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## APPLICATION OF AUTO-TUNING TO A LABORATORY MODEL

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**Abstract:** The paper is focused on application of relay based auto-tuning combined with algebraic controller design to a laboratory heat exchange model. The principle used in this paper consists of two steps. First phase is an identification of the controlled system parameters. It is performed by the relay experiment with biased relay in the feedback loop and consecutive approximation by the first order transfer function with time delay. Second phase is a computation of the controller parameters through parameterized solution of Diophantine equations in the ring of proper and stable rational functions. Controller parameters are tuned through a pole-placement problem as a desired multiple root of the characteristic closed loop equation. This approach enables tuning of the controller parameters by introducing a scalar parameter  $m > 0$  which can be adjusted by several principles.

**Keywords:** auto-tuning, relay, Diophantine equations, feedback control.

### 1 INTRODUCTION

The most used controllers in the industry are still of PID type. They have a simple and understandable structure and they are quite resistant to the control loop changes. However, knowledge of the controlled system parameters is required for the proper controller synthesis. Unfortunately, this requirement is very rarely fulfilled. One of the possible solutions is usage of the auto-tuning procedure. The principle was introduced by Åström in 1984 when a symmetrical relay was used to obtain critical parameters of the controlled system. After it, many studies about the automatic tuning of the controllers are reported (Majhi *et al.* 1998), (Morilla *et al.* 2000), (Vyhliđal 2000), (Vítečková *et al.* 2004).

This contribution is focused on the application of relay based auto-tuning to the real laboratory model. In the first phase, the model is approximated by a first order transfer function with time delay. The following phase is algebraic controller design. Parameters are derived from general solution of the Diophantine equation in the ring of proper and stable rational functions ( $R_{PS}$ ).

### 2 IDENTIFICATION PROCESS

The identification procedure consists of an experiment with biased relay in the feedback loop as can be seen in Fig.1. Typical output from this experiment is shown in Fig.2. The controlled system is then approximated by the first order transfer function with time delay:

$$G(s) = \frac{K}{Ts + 1} \cdot e^{-\Theta s} \quad (1)$$

Estimation of the process gain can be computed from (Vyhliđal 2000):

$$K = \frac{\int_0^{iT_y} y(t) dt}{\int_0^{iT_y} u(t) dt} \quad i = 1, 2, 3, \dots \quad (2)$$

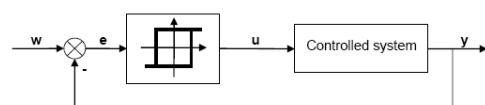


Fig. 1. Relay in the feedback loop

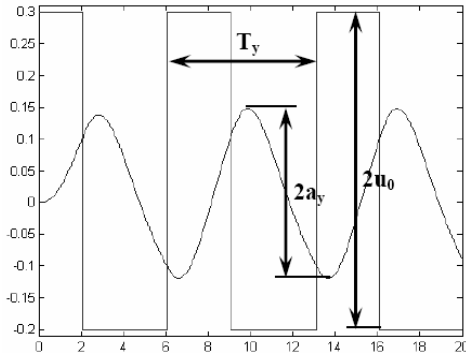


Fig. 2. Output oscillations

The time constant and time delay term are given by (Vítečková *et al.* 2004):

$$T = \frac{T_y}{2\pi} \cdot \sqrt{\frac{16 \cdot K^2 \cdot u_0^2}{\pi^2 \cdot a_y^2} - 1} \quad (3)$$

$$\Theta = \frac{T_y}{2\pi} \cdot \left[ \pi - \arctg \frac{2\pi \cdot T}{T_y} - \arctg \frac{\varepsilon}{\sqrt{a_y^2 - \varepsilon^2}} \right] \quad (4)$$

where  $\varepsilon$  is relay hysteresis.

### 3 CONTROLLER PARAMETERS

The control design is based on the fractional approach (Vidyasagar 1987), (Kučera 1993), (Prokop *et al.* 1997), (Prokop *et al.* 2002). Traditional transfer function as a ratio of two polynomials is transformed into the fractional form by dividing both polynomials by the same stable polynomial of the denominator's order. The conversion between polynomial and  $R_{PS}$  form can be expressed as:

$$G(s) = \frac{b(s)}{a(s)} = \frac{\frac{b(s)}{(s+m)^n}}{\frac{a(s)}{(s+m)^n}} = \frac{B(s)}{A(s)} \quad (5)$$

$$n = \max(\deg(a), \deg(b)), \quad m > 0$$

The time delay term is approximated by the Taylor approximation of the denominator (6) before the controller synthesis. Algebraic design is then performed for the second order transfer function without time delay.

$$e^{-\Theta s} = \frac{1}{e^{\Theta s}} = \frac{1}{1 + \Theta s} \quad (6)$$

All stabilizing controllers are given by the general solution of the Diophantine equation:

$$AP + BQ = 1 \quad (7)$$

which can be expressed by:

$$P = P_0 + BZ \quad Q = Q_0 - AZ \quad (8)$$

where  $P_0$  and  $Q_0$  are particular solutions of (7) and  $Z$  is an arbitrary element of  $R_{PS}$ .

The control aim is not only to stabilize the control loop but also to track the reference value asymptotically and/or reject the disturbances. These requirements are expressed through the divisibility conditions in  $R_{PS}$ . Asymptotic reference tracking is achieved when the denominator  $F_w$  divides  $P$  in  $R_{PS}$ . For the step function it is given by:

$$F_w = \frac{s}{s+m} \quad m > 0 \quad (9)$$

### 4 MODEL DESCRIPTION

Systems with time delay occur in industrial applications very often. Especially in matter transfer systems, e.g. conveyors, pipelines, etc. The laboratory model used in this contribution is the representation of these systems. It is based on the principle of transferring heated liquid through the pipeline to the heat exchanger. Scheme of the model is shown in Fig.3.

The liquid is transferred by continuously controllable pump "6" to the flow heater "1". Temperature of the liquid behind the heater is measured by a platinum thermometer "T1". Heated liquid then flows through a 15 meters long heat-insulated pipeline "2". This pipeline introduces the time delay in the range approximately from 50 to 200 seconds which depends on the pump speed. Next, the liquid flows to the heat exchanger "3". Heat consumption can be adjusted by two fans "4" and "5". Fan "5" can be controlled only in two states (on/off) while fan "4" can be controlled continuously and it is used especially for disturbances generation. Temperature before and behind the heat exchanger is measured by thermometer "T2" and "T3". Finally, the liquid is returned to the pump "6" which creates the media circulation. The tank "7" compensates the water thermal expansion effect.

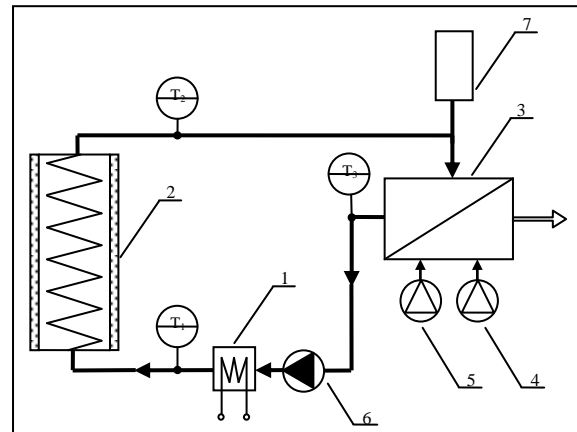


Fig. 3. Scheme of the laboratory model

## 5 EXPERIMENT

The experiment consists of three main parts. At the beginning, the liquid is heated to the constant temperature. After it, the biased relay oscillations are performed. Values depicted in Fig.2 are stored in the computer memory and used for the transfer function approximation. Time delay term is approximated by Taylor approximation of the denominator. Finally, the controller parameters are derived and the control response is recorded.

The output from the experiment is shown in Fig.4. Controlled value in the experiment is temperature of the liquid measured by thermometer “T2”. Sampling period is 10 seconds throughout the whole measurement. All three main parts are clearly noticeable in the figure. The experiment begins with the electric input of the heater set to 30%. It takes some time until the temperature reaches the constant value. Next, the biased relay oscillations are performed. The margins are 20% and 50% of electric input to the heater. Hysteresis of the relay is set to 2°C. After three cycles of the relay, the stored values are used for the transfer function approximation and controller design is performed. The control is started immediately after relay oscillations are stopped. Reference value in the part of control response is set to 60°C.

Approximated transfer function from the relay experiment in Fig.4 is given by:

$$G(s) = \frac{0.88}{343s + 1} \cdot e^{-204s} \quad (9)$$

Transfer function after the time delay approximation is in the form:

$$G(s) = \frac{0.88}{343s + 1} \cdot \frac{1}{1 + 204s} \quad (10)$$

Discrete controller parameters for 10 second sampling period and  $m = 0.0035$  are:

$$\frac{Q(z)}{P(z)} = \frac{0.86z^2 - 1.66z^1 + 0.80}{z^2 - 1.94z^1 + 0.94} \quad (11)$$

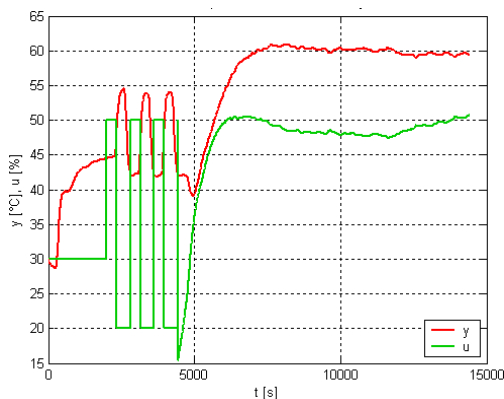


Fig. 4. Relay experiment and control response

## 6 CONCLUSION

This contribution showed the real application of the relay based auto-tuning which was previously studied and simulated in Matlab. It proved that the controlled system parameters can be obtained from the relay experiment as the first order transfer function approximation. The consecutive controller design was performed through the solution of Diophantine equation. This approach introduced the scalar tuning parameter which can be adjusted by several methods. The experiment proved the controller ability to stabilize and control the heat exchange laboratory model.

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