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## OUTPUT STABLE MODEL PREDICTIVE CONTROL DESIGN WITH INPUT CONSTRAINTS

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**Abstract:** The paper addresses the problem to design a quadratic stable output/state feedback model predictive control for linear systems with input constraints. In the proposed design technique the model predictive control is designed for  $N_2$  step ahead prediction using Lyapunov function approach with cost function guaranteeing input constraints. Output gain matrix calculation is realized off line and through dynamic behavior with respect to quadratic stability of closed-loop system only modification of output gain matrix is realized to guarantee input constraints. Two examples are given to demonstrate the effectiveness of the proposed methods.

**Keywords:** model predictive control, quadratic stability, Lyapunov function, input constraints

### 1. INTRODUCTION

Predictive control algorithms compute the control variable minimizing a cost function considering expected future errors in a given prediction horizon. In each sampling time the series of the control signals is calculated, but according to receding horizon strategy only the first value is applied as manipulated variable, and in the next sampling point the procedure is repeated. Predictive algorithms provide good performance especially in case of big dead time and if the future reference trajectory is known. Nowadays in industrial control besides PID control predictive control gains more and more applications. The main criticism related to predictive control is that because of the finite prediction horizon the algorithm in its original formulation does not guarantee stability. Different extensions as e.g. including punishing of end point deviation of the state variables in the cost function, or considering bigger weighting factors for punishing output deviation at the last points of the prediction horizon would ensure stable performance. There are some approaches to guarantee the stability of closed-loop systems. The first type

of approaches has originated by Rawlings and Muske, 1993 where the central idea is that if the minimization problem is feasible the cost function can be interpreted as a monotonically decreasing Lyapunov function and asymptotic stability is therefore guaranteed. The main idea of the next approach, Clarke and Mohtadi, 1989, Clarke and Scattolini, 1991, Dermicioglu and Clarke, 1993 is to impose state terminal constraints to force the predicted output to exactly follow the reference during a sufficiently large horizon. Stability results for constrained MPC have been obtained by Rossiter J.A. and Kouvaritakis B., 1994, who found that for any reference  $w(t)$  which assumes a constant value after a number of sampling periods, if the constrained MPC is feasible for sufficiently large values of the horizon, the closed-loop will be stable. Other approaches the reader can consult in Vesely and Bars, 2008 Rossiter J.A, 2003, Camacho and Bordons, 2004, Maciejowski, 2002, Haber et al., 2002, Mayne et al., 2000. Here closed-loop system stability guarantees are given creating a Lyapunov function for design of state or output feedback controller with input constraints.

The paper is organized as follows. The next sec-

tion gives a problem formulation and preliminary about a predictive output/state model. In Section 3, the approach of output feedback predictive controller design using linear matrix inequality is presented. In Section 4, the input constraints are formulated to LMI feasible solution. Two examples in Section 5 illustrate the effectiveness of the proposed method. Finally, some conclusions are given.

## 2. PRELIMINARIES AND PROBLEM FORMULATION

A time invariant linear discrete-time system is given by

$$\begin{aligned} x(t+1) &= Ax(t) + Bu(t) \\ y(t) &= Cx(t) \end{aligned} \quad (1)$$

where  $x(t) \in R^n, u(t) \in R^m, y(t) \in R^l$  are state, control and output variables of the system, respectively;  $A, B, C$  are known matrices of corresponding dimensions.

The state of the model for instant  $t + N_2$  can be computed recursively, (Camacho and Bordons, 2004) applying (1), the result is:

$$\begin{aligned} x(t + N_2) &= A^{N_2}x(t) + \\ &\sum_{i=0}^{N_2-1} A^{N_2-i-1}Bu(t+i) \end{aligned} \quad (2)$$

and the corresponding output is

$$y(t + N_2) = Cx(t + N_2) \quad (3)$$

Consider a set of  $N_2$  steps ahead predictions, one obtains

$$\begin{aligned} z(t+1) &= A_f z(t) + B_f v(t) \\ y_f(t) &= C_f z(t) \end{aligned} \quad (4)$$

where

$$\begin{aligned} z(t)^T &= [x(t)^T \ x(t+1)^T \ \dots \ x(t+N_2-1)^T] \\ v(t)^T &= [u(t)^T \ u(t+1)^T \ \dots \ u(t+N_2-1)^T] \\ y_f(t)^T &= [y(t)^T \ y(t+1)^T \ \dots \ y(t+N_2-1)^T] \\ A_f &= \begin{bmatrix} A & 0 & \dots & 0 \\ A^2 & 0 & \dots & 0 \\ \dots & \dots & \dots & \dots \\ A^{N_2} & 0 & \dots & 0 \end{bmatrix} \in R^{(nN_2) \times (nN_2)} \\ B_f &= \begin{bmatrix} B & 0 & \dots & 0 \\ AB & B & \dots & 0 \\ \dots & \dots & \dots & \dots \\ A^{N_2-1}B & \dots & \dots & B \end{bmatrix} \in R^{(nN_2) \times (mN_2)} \end{aligned}$$

$$C_f = \begin{bmatrix} C & 0 & \dots & 0 \\ 0 & C & \dots & 0 \\ \dots & \dots & \dots & \dots \\ 0 & 0 & \dots & C \end{bmatrix} \in R^{(lN_2) \times (nN_2)}$$

If the state  $x(t)$  for state feedback is not accessible, Kalman filter is required.

Let the cost function to be minimized be in the form (Kothare et al. 1996)

$$J = \sum_{t=t_0}^{\infty} J(t) \quad (5)$$

where

$$\begin{aligned} J(t) &= \min_{v(t)} \{ [y_f(t+1) - w_f(t+1)]^T Q \\ &[y_f(t+1) - w_f(t+1)] + v(t)^T R v(t) \} \end{aligned}$$

and

$$w_f(t+1)^T = [w(t+1)^T \ \dots \ w(t+N_2)^T]$$

gives the predicted setpoints.  $Q, R$  are positive definite matrices.

The problem studied in this part of paper is to design model predictive controller with output (state) feedback in the form

$$v(t) = F(y_f(t) - w_f(t)) \quad (6)$$

where  $F$  is output feedback gain matrix which guarantees stability of closed-loop system and minimum value of the cost function  $J(t)$  subject to (5).

## 3. MODEL PREDICTIVE CONTROLLER DESIGN

Let us consider a set of  $y_f(t+j)$   $j \in \{1, \dots, N_2\}$  ahead output predictions affecting the cost function (5) and the vector of  $v(t+j)$ ,  $j \in \{0, \dots, N_u\}$  future control in the control horizon. A new system and input matrices are formed by the corresponding submatrices of  $A_f, B_f \in R^{(nN_2) \times (mN_u)}$  respectively. For more detail see, Camacho and Bordons, 2004. We will proceed without changing the denotation.

$$\begin{aligned} v(t) &= F(y_f(t) - w_f(t)) = \\ &F(C_f z(t) - w_f(t)) \end{aligned} \quad (7)$$

and the closed-loop system is given by

$$\begin{aligned} z(t+1) &= (A_f + B_f F C_f) z(t) - B_f F w_f(t) \\ &= A_c z(t) - B_f F w_f(t) \end{aligned} \quad (8)$$

Because the vector  $w_f(t)$  is independent from vector  $z(t)$  and if vector  $w_f(t)$  belongs to the class

of  $L_2$  the stability of the closed-loop system (8) is determined by matrix  $A_c$ . The origin of the state vector  $z(t)$  has to be recalculated to a new steady state given by the set point vector  $w_f(t)$ . Due to Lyapunov function approach and because the stability of closed-loop system determined by the matrix  $A_c$  we assume that  $w(t) = w(t+1) = \dots = 0$ . The closed-loop system will be stable if and only if the first difference of Lyapunov function  $V(t) = z(t)^T P z(t)$ ,  $P = P^T > 0$  on the solution of (8) will be negative definite (semidefinite) that is

$$\Delta V(t) = V(t+1) - V(t) = \quad (9)$$

$$z(t)^T (A_c^T P A_c - P) z(t) \leq 0 \quad t = 1, 2, \dots$$

Substituting (7) and (8) to cost function (5) one obtains

$$J(t) = \min_F z(t)^T (A_c^T C_f^T Q C_f A_c + \quad (10)$$

$$C_f^T F^T R F C_f) z(t) \geq 0$$

From (10) one can see that  $J(t)$  for  $t = 1, 2, \dots$  is positive definite (semidefinite) function. Closed-loop system (8) will be stable and cost function  $J(t)$  reaches minimal value iff the following inequality holds (Krokavec and Filasova, 2008)

$$B_e(t) = z^T(t) [A_c^T P A_c - P + \quad (11)$$

$$A_c^T C_f^T Q C_f A_c + C_f^T F^T R F C_f] z(t) \leq 0$$

for  $t = 1, 2, \dots$  e.i. the matrix expression in (11) will be negative definite (semidefinite). From (11) one can recast the following bilinear matrix inequality (BMI)

$$\begin{bmatrix} -P & A_c^T & A_c^T C_f^T Q & C_f^T F^T R \\ * & -P^{-1} & 0 & 0 \\ * & * & -Q & 0 \\ * & * & * & -R \end{bmatrix} \leq 0 \quad (12)$$

where \* denote the transpose of corresponding symmetric part of matrix. Using linearization approach de Oliveira et al. 2000 with respect to  $P^{-1}$  the following inequality holds

$$-P^{-1} \leq Y_k^{-1} (P - Y_k) Y_k^{-1} - Y_k^{-1} = \quad (13)$$

$$= \text{lin}(-P^{-1})$$

Due to (13), (12) becomes LMI in the form

$$\begin{bmatrix} -P & A_c^T & A_c^T C_f^T Q & C_f^T F^T R \\ * & \text{lin}(-P^{-1}) & 0 & 0 \\ * & * & -Q & 0 \\ * & * & * & -R \end{bmatrix} \leq 0 \quad (14)$$

where  $Y_k, k = 1, 2, \dots$  in the iteration process  $Y_k = P$ . Because the linearization approach have

been used for (12), BMI conditions (12) "if and only if" reduces to "if" for (14). If the LMI (14) is feasible with respect to matrices  $P$  and  $F$ , closed-loop system will be quadratically stable with minimal value of the cost function. Note that using (7) with matrix  $F$  one could calculate the input (control) vector  $v(t)$  for  $N_u - 1$  predicted control horizon from  $N_2$  predicted output vector  $y_f(t)$ . As a receding horizon strategy is used, only the first element of the control vector  $u(t)$  is sent to the plant and all the computation is repeated at the next sampling time. Let us rewrite (4) as follows

$$z(t+1) = A_f z(t) + (B_1 + B_2)v(t) \quad (15)$$

where  $B_f = B_1 + B_2$   $B_1 = [B_{f1} \quad 0_1]$   $B_2 = [0_2 \quad B_{f2}]$  and

$$B_{f1} \in R^{(nN_2) \times m} \quad 0_1 \in R^{(nN_2) \times (N_u - 1)m}$$

$$B_{f2} \in R^{(nN_2) \times (N_u - 1)m} \quad 0_2 \in R^{(nN_2) \times m}$$

and  $0_i, i = 1, 2$  are zero matrices with corresponding dimensions.

From (15) we get

$$(B_1 + B_2)v(t) = B_{f1}u(t) + O_1v(t+1) + \quad (16)$$

$$O_2u(t) + B_{f2}v(t+1)$$

Equation (16) implies: to guarantee the stability of closed-loop system for control variable  $u(t)$  instead of matrix  $B_f$  in (14) one has to use the matrix of  $B_1$ .

We can conclude that if the following two LMIs:

$$\{\text{ineq}(14) \text{ with } B_f\} \leq 0 \quad (17)$$

$$\{\text{eq}(14) \text{ with } B_1\} \leq 0$$

are feasible with respect to  $P = P^T \Rightarrow 0$  and matrix  $F$ , the closed-loop system with control variables  $v(t)$  and  $u(t)$  will be quadratically stable and the cost function will have minimal value. Note that only the first  $m$  rows of matrix  $F$  are used for real plant control with control input  $u(t)$ . The above results can be summarized in the following theorem.

*Theorem 1.*

We are given a discrete linear-time invariant system (1). Assume that control algorithm for model predictive control is given by (6), where  $w(t+j) = 0$  or constant,  $j = 1, 2, \dots, N_u - 1$ . (For the case of  $w(t+j) = \text{cons}$  the origin of state space of system (1) have to be recalculated to achieve the steady state given by constant set points). If the two LMIs (17) are feasible with respect to  $P = P^T > 0$  and a matrix  $F$ , then cost function

(6) has minimal value and quadratic stability of closed-loop system is guaranteed for two cases

- output feedback with control algorithm given by (6) and
- output feedback for control variable  $u(t) = Fy_f(t)$  when only first  $m$  rows of matrix  $F$  are used

#### 4. MPC DESIGN FOR INPUT CONSTRAINTS

To design model predictive control (Adamy and Flemming, 2004, Camacho and Bordons, 2004) with any constraints on input, state and output variables at each sampling time, starting from the current state, an open-loop optimal control problem is solved over the defined finite horizon. The first element of the optimal control sequence is applied to the plant. At the next time step, the computation is repeated. Thus, the implementation of the MPC strategy requires a QP solver for the on-line optimization which still requires significant on-line computational effort, which limits MPC applicability. In this paper we propose the off-line calculation of two control gain matrices and using analogy to SVSS approach, Adamy and Flemming, 2004 we significantly reduce the computational effort for MPC suboptimal control with input constraints.

Consider the system (4) where the control  $v(t)$  is constrained to evolve in the following set

$$\Gamma = \{v \in R^{mN_2} : |v_i(t)| \leq U_i, i = 1, \dots, mN_2\} \quad (18)$$

The aim of this part of paper is to design the stabilizing control output feedback law for system (4) in the form

$$v(t) = FC_f z(t) \quad (19)$$

which guarantees that for the initial state  $z_0 \in \Omega(P) = \{z(t) : z(t)^T P z(t) \leq \theta\}$  control  $v(t)$  belongs to the set (18) for all  $t \geq 0$ , where  $P = P^T > 0$  is positive definite matrix,  $\theta$  is positive real parameter which determines the size of  $\Omega(P)$ . Furthermore,  $\Omega(P)$  should be such that all  $z(t) \in \Omega(P)$  provide  $v(t)$  satisfying the relation (18), restricting the values of the control parameters. Moreover, the following ellipsoidal Ljapunov function level set

$$\Omega(P) = \{z(t) \in R^{nN_2} : z(t)^T P z(t) \leq \theta\} \quad (20)$$

can be proven to be a robust positively invariant region with respect to motion of the closed-loop system in the sense of the following definition, Rohal-Ilkiv, 2004, Ayd and et al, 2008.

*Definition* A subset  $S \in R^{(nN_2)}$  is said to be positively invariant with respect to motion of system (4) with control algorithm (7) if for every

initial state  $z(0)$  inside  $S$  the trajectory  $z(t)$  remains in  $S$  for all  $t \geq 0$ .

Consider that vector  $F_i$  denotes the  $i$ -th row of matrix  $F$  and define

$$L(F) = \{z(t) \in R^{(nN_2)} : |F_i C_f z(t)| \leq U_i, \} \\ i = 1, 2, \dots, mN_2$$

The above set can be written as

$$L(F) = \{z(t) \in R^{(nN_2)} : |D_i F C_f z(t)| \leq U_i, \} \quad (21) \\ i = 1, 2, \dots, mN_2$$

where  $D_i \in R^{1 \times mN_2} = \{d_{ij}\}, d_{ij} = 1, i = j, d_{ij} = 0, i \neq j$ . The results are summarized in the following theorem.

*Theorem 2* The inclusion  $\Omega(P) \in L(F)$  is for output feedback control equivalent to

$$\begin{bmatrix} P & C^T F^T D_i^T \\ D_i F C & \lambda_i \end{bmatrix} \geq 0, i = 1, 2, \dots, mN_2 \quad (22)$$

where  $\lambda_i \in < 0, \frac{U_i^2}{\theta} >$ .

*Proof.* To prove the inclusion  $\Omega(P) \in L(F)$  is equivalent to (22) we use  $S$ - procedure in following way. Rewrite (20) and (21) in the following form

$$p(z) = z^T(t) P z(t) - \theta \leq 0$$

$$g_i(z) = z^T(t) C_f^T F^T D_i^T D_i F C z(t) - U_i^2 \leq 0$$

According to  $S$ - procedure there exists a positive scalar  $\lambda_i$  such that

$$g_i(z) - \lambda_i p(z) \leq 0$$

or equivalently

$$z(t)^T (C_f^T F^T D_i^T D_i F C - \lambda_i P) z(t) - U_i^2 + \lambda_i \theta \leq 0 \quad (23)$$

Taking any scalar  $\rho \neq 0$  and multiplying (23) from left and right side we obtain

$$\begin{bmatrix} C_f^T F^T D_i^T D_i F C - \lambda_i P & 0 \\ 0 & -U_i^2 + \lambda_i \theta \end{bmatrix} \leq 0 \quad (24) \\ i = 1, 2, \dots, mN_2$$

The above diagonal matrix is equivalent to two inequalities. Using Schur complement formula for the first one one obtains the inequality (22) which proves the theorem.

In order to check the value of  $\theta_i$  for  $i$ -th input one solves the optimization problem  $z(t)^T P z(t) \rightarrow \max$  subject to constraints (18), whose solution yields

$$\theta_i = \frac{U_i^2}{D_i F C P^{-1} C^T F^T D_i^T}$$

In the design procedure it should be verified that when parameter  $\theta$  decreases the obtained robust

positively invariant regions  $\Omega(P)$  are nested to region obtained for  $\theta + \epsilon, \epsilon > 0$ .

Assume that one calculates two output feedback gain matrices  $F_1$  for unconstrained case and  $F_2$  for constrained one. Normally, closed-loop system with the gain matrix  $F_2$  gives the dynamic behavior slower than one obtain for  $F_1$ . Consider the resulting output feedback gain matrix  $F$  in the form

$$F = \gamma F_1 + (1 - \gamma)F_2, \gamma \in (0, 1) \quad (25)$$

For gain matrices  $F_i, i = 1, 2$  one obtains two closed-loop system in the form (8),  $A_{ci} = A_f + B_f F_i C_f, i = 1, 2$ . Consider the edge between within  $A_{c1}$  and  $A_{c2}$ , that is

$$A_c = \alpha A_{c1} + (1 - \alpha)A_{c2}, \alpha \in (0, 1) \quad (26)$$

The following lemma gives the stability conditions for matrix  $A_c$  (26).

*Lemma 1* Consider the stable closed-loop matrices  $A_{ci}, i = 1, 2$ .

- If there exists a positive definite matrix  $P_q$  such that

$$A_{ci}' P_q + P_q A_{ci} \leq 0, i = 1, 2 \quad (27)$$

then matrix  $A_c$  is quadratically stable.

- If there exist two positive definite matrices  $P_1, P_2$  such that they satisfy the stability conditions given by Grman et al, 2005 the closed-loop system  $A_c$  is parameter dependent quadratically stable (PDQS).

*Remarks*

- If the closed-loop matrices  $A_{ci}, i = 1, 2$  satisfy (27) the scalar  $\gamma$  in (25) may be change with any rate without violating the closed-loop stability.
- If the closed-loop matrices  $A_{ci}, i = 1, 2$  are only PDQS, the scalar  $\gamma$  in (25) has to be constant but may be unknown.
- The proposed control algorithm (25) is similar to Soft Variable-Structure Control (SVSC) Adamy and Flemming, 2004 but in our case, when  $v_i < U_i$  the feedback gain matrix  $F$  (25) gives rather more quick dynamic behavior to the closed-loop system then when  $v_i$  approaches  $U_i$ .

Algorithm for calculation of  $\gamma$  (25) may be as follows:

$$\gamma = \min_i \frac{U_i - |v_i|}{U_i}$$

If accidentally some  $v_i > U_i, \gamma = 0$ .

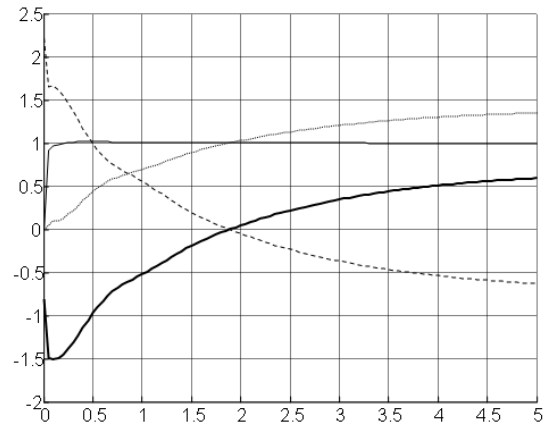


Figure 1. Dynamic behavior of unconstrained controlled system

### 5. EXAMPLES

First example has been borrowed from Camacho and Bordons, 2004 p.147. The model corresponds to the longitudinal motion of a Boeing 747 airplane. The multivariable process is controlled using a predictive controller based on the output model of the aircraft. Two of the usual command outputs that must be controlled are airspeed that is, velocity with respect to air, and climb rate. Continuous model has been converted to discrete time one with sampling time of 0.1s, the model turns to (1) where

$$A = \begin{bmatrix} .9996 & .0383 & .0131 & -.0322 \\ -.0056 & .9647 & .7446 & .0001 \\ .002 & -.0097 & .9543 & 0 \\ .0001 & -.0005 & .0978 & 1 \end{bmatrix}$$

$$B = \begin{bmatrix} .0001 & .1002 \\ -.0615 & .0183 \\ -.1133 & .0586 \\ -.0057 & .0029 \end{bmatrix} \quad C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 7.74 \end{bmatrix}$$

Note that matrix  $A$  is unstable. For the weighted matrices  $Q = 10I, R = I, N_u = N_2 = 6$  the following results are obtained:

- Unconstrained case  $Closed - loopmaxeig = 0.8371 \pm 0.2873i$
- Constrained case with  $U_i = 1, \theta = 1$   $Closed - loopmaxeig = 0.7546 \pm 0.3010i$

Closed-loop step responses of the above two cases are given in the Fig.1 and Fig.2. The closed-loop system (26) is quadratically stable. Second example serve as a benchmark. The model of double integrator turns to (1) where

$$A = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$$

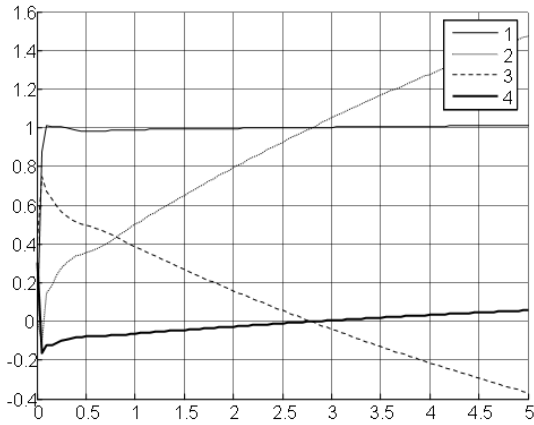


Figure 2. Dynamic behavior of constrained controlled system. 1,2 outputs, 3,4 inputs

$$B = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, C = [0 \ 1]$$

For the case of  $N_2 = 6, N_u = 6, Q = 5I, R = I$  for unconstrained case the gain matrix to control variable  $v(t) = F1y_f(t)$  is as follows

$$F1 = \begin{bmatrix} -0.0206 & 0.0097 & -0.0081 & -0.0177 \\ -0.6432 & 0.0057 & -0.0052 & -0.0111 \\ 0.4136 & -0.0038 & 0.0033 & 0.0072 \\ 0.1700 & -0.0016 & 0.0014 & 0.0029 \\ 0.0582 & -0.0005 & 0.0005 & 0.0010 \\ 0.0310 & -0.0002 & 0.0003 & 0.0005 \end{bmatrix}$$

$$\begin{bmatrix} -0.0241 & -0.0293 \\ -0.0151 & -0.0183 \\ 0.0097 & 0.0118 \\ 0.0040 & 0.0049 \\ 0.0014 & 0.0017 \\ 0.0007 & 0.0009 \end{bmatrix}$$

and for constrained case with  $U_i = 0.3, i = 1, 2, \dots, 6, \theta = 1$

$$F2 = \begin{bmatrix} 0.0441 & 0.0600 & -0.0111 & -0.0476 \\ -0.2271 & 0.0182 & -0.0014 & -0.0112 \\ 0.2389 & -0.0163 & 0.0023 & 0.0119 \\ 0.0947 & -0.0065 & 0.0009 & 0.0047 \\ 0.0407 & -0.0028 & 0.0004 & 0.0020 \\ 0.0266 & -0.0018 & 0.0003 & 0.0013 \end{bmatrix}$$

$$\begin{bmatrix} -0.0707 & -0.0881 \\ -0.0171 & -0.0215 \\ 0.0178 & 0.0222 \\ 0.0071 & 0.0088 \\ 0.0030 & 0.0038 \\ 0.0020 & 0.0025 \end{bmatrix}$$

Closed-loop step responses for unconstrained and constrained cases are given in Fig.3 and Fig.4. The closed-loop system (26) is quadratically stable.

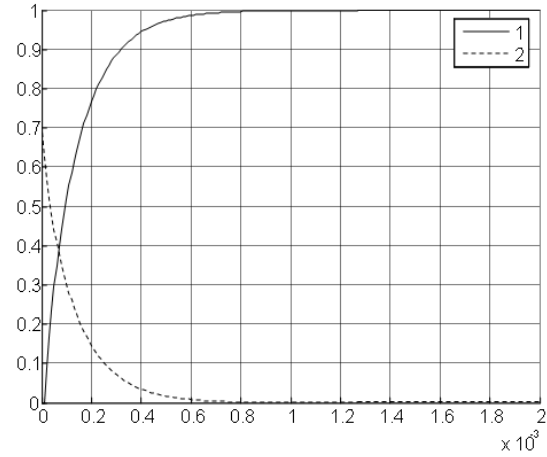


Figure 3. Dynamic behavior of controlled system for unconstrained case. 1-output,2 input

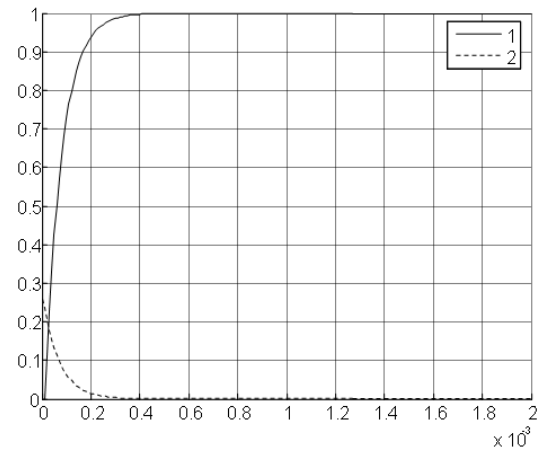


Figure 4. Dynamic behavior of controlled system for constrained case. 1-output, 2-input

## 6. CONCLUSION

The paper addresses the problem of the design of state or output feedback model predictive controller with input constraints for  $N_2$  step prediction. The obtained control algorithm guarantees the closed-loop system quadratic stability and optimal value of performance function using Lyapunov function approach. Finally two examples are given which shows the effectiveness of proposed methods.

## 7. ACKNOWLEDGMENT

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