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# APPROXIMATE MATHEMATICAL MODEL OF A STEAM OVERHEATER

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Abstract: We describe a construction of a mathematical model of a controlled power-plant steam overheater. The overheater consists of two subsystems that can be identified separately – cooling steam by water injection and the heating steam part. The model is subsequently to be used in design of a cascade-type control system for regulation of outlet temperature of overheated steam. The second subsystem has distributed parameters and for the purposes of control design it has to be simplified. One way of such an approximation is proposed in the paper. The procedure of estimation of the model parameters from measured service data is described in a stand-alone paper in the same proceedings.

Keywords: heat systems, mathematical modeling, system identification, control systems.

## 1 INTRODUCTION

A practical demand for most steam boilers is keeping temperature of overheated steam on a desired level. For powerplant boilers there exists an optimal value of temperature, for which maximal production gain is obtained, because with rising temperature of steam there increase both efficiency of the turbine and the equipment renewal costs [Karták *et al*. 1981]. Steam is heated in the steam overheater. Outlet steam temperature is influenced by the temperature of steam in the input of the overheater, steam flow rate and heat input. As usual, only ending part of whole overheater is controlled. In principle, regulation can be carried out by heat input change, by a change of input temperature by cold water injection or by cooling in a surface exchanger. Here the second variant is considered, which is also the most common case in practice (Fig. 1). The cooler is constructed so that a sufficient dispersion of water drops that must not come into the heating part is achieved.

From the service data obtained from one of controlled overheaters in the powerplant in Opatovice nad Labem (International Power Opatovice, a.s.) it was observed that although the temperature is automatically controlled, the variance of $T_3$ is often significantly bigger than the variance of $T_1$ in the input. This behaviour can be caused by strong external influences, but also by improper settings of the control loop parameters. The paper describes a construction

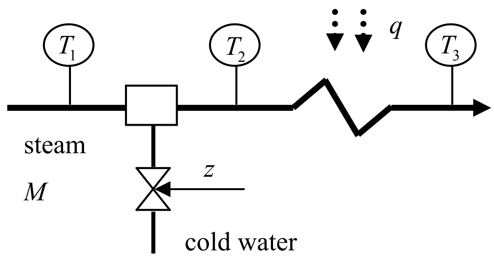


Fig. 1. Control of the temperature of overheated steam by cold water injection

of a mathematical model of the system, which should enable enhancements in the control system design.

Disturbances in effect are the changes of the temperature $T_1$, the flow rate $M$ and the heat input $q$. The temperature $T_2$ is easily controllable by the input valve control variable $z$, because at the same time the temperature $T_1$ and the flow rate $M$ are measured. However, regulation of the outlet temperature is difficult, because due to the overheater length a significant time delay occurs between input point and the output. Therefore, cascade configuration of two controllers is usually used, where inner loop shall compensate or speed up dynamics of mixing and outer loop ensures control of temperature by means of change of $T_2$ (Fig. 2).

The control loop parameters can be designed on the basis of a mathematical model of the controlled plant. Obtaining the model structure using mathematical-physical analysis is described below. We assume that the model approximate parameter values can be obtained by processing measured service data. Parameter estimation procedure is described in a stand-alone paper published in the same proceedings [Cvejn 2009].

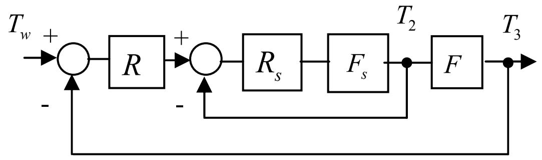


Fig. 2. Cascade regulation of outlet temperature

## 2 MATHEMATICAL MODEL OF THE OVERHEATER

### 2.1 Model of the cooler

The subsystem of cooling steam by cold water injection was modeled by a single capacity dynamic plant. Denote $V$ internal volume of the cooler, $\rho$ density of steam, $c_p, c_v$ specific heat capacities of steam and water, $T_v$ temperature of cooling water, $T_b$ boiling point of water, $C_v$ latent heat of water, $M_p$ flow rate of steam before injection and $M_v$ flow rate of cooling water through the control valve. After substitution into the heat balance we obtain the equation

$$\begin{aligned}\rho V c_p \frac{dT_2}{dt} &= -M_p c_p \left(T_2 - T_1\right) \\ &- M_v \left[c_v \left(T_b - T_v\right) + c_p \left(T_2 - T_b\right) + C_v\right].\end{aligned} \tag{1}$$

By expressing linearized deviations from steady state we obtain the equation

$$\begin{aligned}\rho V c_p \frac{d\vartheta_2}{dt} &+ \left(M_p + M_v\right) c_p \vartheta_2 = \\ &= M_p c_p \vartheta_1 + c_p \left(T_1 - T_2\right) \xi_p + \\ &+ \left[c_v \left(T_v - T_b\right) + c_p \left(T_b - T_2\right) - C_v\right] \xi_v\end{aligned} \tag{2}$$

where $\vartheta_1$, $\vartheta_2$ are the changes of temperature in the input and the output of the cooler, $\xi_p$, $\xi_v$ the changes of the steam inlet flow and the cooling water flow. In eq. (2) symbols $T_1, T_2, M_v$ and $M_p$ denote the steady-state values of quantities. For considered equipment holds $M_v = M_{v\max} z$, where $M_{v\max}$ is maximal flow rate given by the cooling water pressure and $z \in \langle 0, 1 \rangle$ the control variable of the valve. For the inlet flow $\xi_v$ and overall change of the flow rate $\xi$ holds $\xi_v = M_{v\max} u$ and $\xi = \xi_p + M_{v\max} u$, respectively, where $u$ is the change of the valve control variable. After substitution into equation (2) we obtain

$$\begin{aligned}\rho V c_p \frac{d\vartheta_2}{dt} + M c_p \vartheta_2 &= M_p c_p \vartheta_1 + c_p \left(T_1 - T_2\right) \xi + \\ + M_{v\max} &\left[c_v \left(T_v - T_b\right) + c_p \left(T_b - T_1\right) - C_v\right] u.\end{aligned} \tag{3}$$

From equation (3) it is possible directly to express transfer functions in Laplace transform:

$$F_T(s) = \frac{K_T}{T_s s + 1}, F_M(s) = \frac{K_M}{T_s s + 1}, F_z(s) = \frac{K_z}{T_s s + 1} \tag{4}$$

where $K_T$, $K_M$, $K_z$ are the gains corresponding to changes of the temperature of inlet steam, the flow and the control variable of the valve. $T_s$ is common cooling time constant.

### 2.2 Model of the steam heater

We assume that heat input is constant along the overheater. Given equipment uses combined way of transferring heat – by convection and by radiation. By a suitable construction of the overheater it is possible to achieve that the heat input is only a little dependent on the flow rate [Karták *et al.* 1981]. Since this fact usually has been used in design of overheaters, we consider that dependence of the heat transfer coefficient on the flow rate can be neglected.

If we denote $x$ the distance of the point of measuring the steam temperature $T$ from the beginning of the heating part, the temperature is besides time also dependent on the coordinate $x$. For the element of steam of length $dx \to 0$ it is possible to write the balance equation

$$\rho c S dx \frac{dT}{dt} = Mc\left[T(x) - T(x + dx)\right] + \alpha O dx (T_t - T) \tag{5}$$

where $S$ is inner section of the pipe, $O$ inner perimeter of the pipe, $c$ heat capacity of steam, $\rho$ steam density, $M$ mass flow rate of steam, $\alpha$ the coefficient of the heat transfer between the pipe and steam, $T$ and $T_t$ the temperatures of steam and the pipe (since the overheater pipe is thin, it is possible to consider that its temperature is constant in the section). After division by $dx$ and expressing the changes of the temperatures $\vartheta$, $\vartheta_t$ and the flow rate $\xi$ in linearized form we obtain the partial differential equation

$$\rho c S \frac{\partial \vartheta}{\partial t} + Mc \frac{\partial \vartheta}{\partial x} = \alpha O(\vartheta_t - \vartheta) - cT_x \xi \qquad (6)$$

where $T_x(x) = \partial T / \partial x$ does not depend on time.

For the element of the pipe length we obtain similarly (heat input is independent of *x*, but is dependent on time)

$$Gc_t \frac{\partial \vartheta_t}{\partial t} = qO - \alpha O(\vartheta_t - \vartheta) \qquad (7)$$

where $G$ is the weight of unit of the pipe length, $c_t$ the heat capacity of the pipe material, $q$ the heat input taken for unit of the area of inner face of the pipe. Since $\vartheta_t(x,t)$ is zero for $t = 0$ and $q$ does not depend on *x*, after Laplace transform it is possible to express the image $\Theta_t(x,s)$ in the form

$$\Theta_t(x,s) = \frac{1}{\tau_t s + 1}\left[\frac{1}{\alpha} Q(s) + \Theta(x,s)\right] \qquad (8)$$

where $\tau_t = Gc_t / (\alpha O)$ and $s$ is the Laplace operator. If we suppose that $\vartheta(x,0) = 0$, after substitution of $\Theta_t(x,s)$ into the image of eq. (6) and rearrangement we obtain

$$\frac{\partial \Theta(x,s)}{\partial x} + \frac{1}{Mc}\left[\rho c S s + \alpha O \frac{\tau_t s}{\tau_t s + 1}\right]\Theta(x,s) = R(s) \qquad (9)$$

where

$$R(s) = \frac{O}{Mc}\frac{1}{\tau_t s + 1} Q(s) - \frac{1}{M} T_x \Xi(s) . \qquad (10)$$

In the case when $R(s) = 0$, the solution of this equation with respect to *x* is

$$\Theta(x,s) = e^{-\frac{1}{Mc}\left[\rho c S s + \alpha O \frac{\tau_t s}{\tau_t s + 1}\right] x} \Theta(0,s) . \qquad (11)$$

For $x = L$, where $L$ is the overheater length, we obtain

$$\Theta(L,s) = \left( e^{-T_d s} e^{-\kappa \frac{\tau_t s}{\tau_t s + 1}} \right) \Theta(0,s) = F_g(s).\Theta(0,s) \qquad (12)$$

where $T_d = \rho S L / M$ has meaning of transport delay and

$$\kappa = \frac{\alpha O L}{Mc} . \qquad (13)$$

Since $\vartheta(0,t)$ represents the system input, $F_g(s)$ is the transfer function between $T_2$ and $T_3$.

The second part of the transfer function

$$\Gamma(s) = e^{-\kappa \frac{\tau_t s}{\tau_t s + 1}} \qquad (14)$$

cannot be realized by a system with concentrated parameters. By inspecting the limit $s \to 0$ we easily find out that the corresponding static gain equals 1. Step responses corresponding to $\Gamma(s)$ can be found in the literature [Čermák *et al*. 1968]. The transfer function $F_g(s)$ can be replaced by a transfer function of a higher-order plant that can be obtained from Taylor expansion [Čermák *et al*. 1968]. However, since the transport delay $T_d$ is assumed to be important, in this case we obtain a high-order plant, which is not advantageous for further manipulation. Therefore, we believe that leaving transport delay in the transfer function and approximating $\Gamma(s)$ by a rational function can be a more advantageous approach. A special choice is Padè approximation, which is given by agreement of derivatives in the origin. It is known, e.g. [Smith 2006], that for the function $e^{-z}$ Padè approximant of the second order is in the form

$$e^{-z} \approx \frac{1 - z/2}{1 + z/2} . \qquad (15)$$

Substituting $z = \kappa \frac{\tau_t s}{\tau_t s + 1}$ we obtain

$$\Gamma(s) \approx \frac{(1 - \kappa/2)\tau_t s + 1}{(1 + \kappa/2)\tau_t s + 1} = \frac{\tau_1 s + 1}{\tau_2 s + 1} . \qquad (16)$$

If exact agreement of derivatives in the origin is not important, by a suitable choice of the constants $\tau_1$, $\tau_2$ a better approximation can be achieved in the whole interval $[0, \infty)$. By comparison of frequency responses it was observed that the model is a good approximation of $\Gamma(s)$ if $\kappa < 1$. See the figures 3 and 4 for comparison of real and imaginary part of $\Gamma(i\omega)$ (solid line) and its Padè approximation (15) (dashed line), for $\tau_t = 1s$ and $\kappa = 1$.

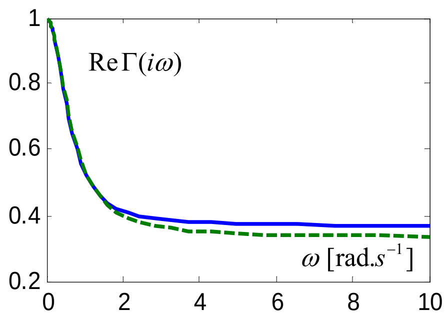


Fig. 3. Approximation of $\Gamma(i\omega)$ - real part

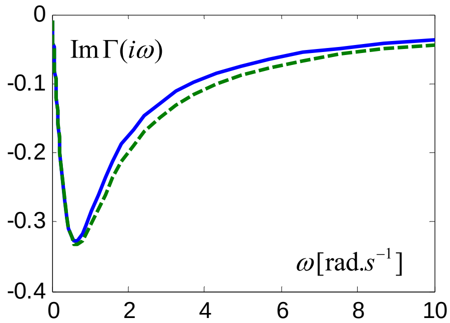


Fig. 4. Approximation of $\Gamma(i\omega)$ - imaginary part

For $\kappa > 2$ the model is not satisfactory. This is also indicated by the time constant $\tau_1$ in (16), which is negative for $\kappa > 2$. Since $\kappa$ plays the role of an exponent in $\Gamma(s)$, an adequate model in this case is

$$\Gamma(s) \approx \left(\frac{\tau_1 s+1}{\tau_2 s+1}\right)^p \tag{17}$$

where $p \ge 1$ is the integer number nearest to $\kappa$.

The conclusion is that we suggest that the transfer function be approximately modeled as

$$F_\vartheta(s) = \left(\frac{\tau_1 s+1}{\tau_2 s+1}\right)^p e^{-T_d s} \tag{18}$$

where $\tau_1, \tau_2, T_d$ are some positive time constants and $p$ suitable integer parameter.

Under a simplifying assumption that $T_x(x) = \partial T / \partial x$ does not much depend on $x$ we obtain dependence between $R(s)$ and $\vartheta_3(s)$ by solving the equation (9) with zero initial condition $\Theta(0,s) = 0$:

$$\begin{aligned}\Theta(x,s) &= \int_0^L e^{-\frac{1}{Mc}\left[\rho c S s + \alpha O \frac{\tau_t s}{\tau_t s+1}\right]x} R(s)\,dx = \\ &= \left[1 - F_\vartheta(s)\right]\Phi(s)R(s)\end{aligned} \tag{19}$$

where

$$\begin{aligned}\Phi(s) &= \frac{1}{\frac{1}{Mc}\left[\rho c S s + \alpha O \frac{\tau_t s}{\tau_t s+1}\right]} = \\ &= \frac{1}{s}\frac{Mc(\tau_t s+1)}{\rho c S(\tau_t s+1) + \alpha O \tau_t}.\end{aligned} \tag{20}$$

With regards to (10) the transfer functions between $q$ and $\vartheta_3(s)$ or $\xi$ and $\vartheta_3(s)$, respectively, are after rearrangement

$$F_q(s) = \frac{O}{\rho c S + \alpha O \tau_t}\,\frac{1}{\frac{\rho c S}{\rho c S + \alpha O \tau_t}\tau_t s + 1} H(s) \tag{21}$$

$$F_\xi(s) = -\frac{c T_x}{\rho c S + \alpha O \tau_t}\,\frac{\tau_t s + 1}{\frac{\rho c S}{\rho c S + \alpha O \tau_t}\tau_t s + 1} H(s) \tag{22}$$

where

$$H(s) = \frac{1}{s}\left[1 - \left(\frac{\tau_1 s+1}{\tau_2 s+1}\right)^p e^{-T_d s}\right]. \tag{23}$$

The transfer function $H(s)$ can be approximately replaced if we consider the shape of corresponding step response, which has constant derivative of 1 in the interval $[0, T_d]$, but since the point $t = T_d$ the slope decreases and for $t \to \infty$ it is zero. Fig. 5 shows the step response corresponding to the transfer function for $\tau_1 = 0.1$, $\tau_2 = 1$, $T_d = 2$ and $p = 1$ (solid line). Approximation with the 1st-order plant in the form

$$H(s) \approx \frac{K_H}{\tau_H s + 1} \tag{24}$$

(dashed line) is obviously possible.

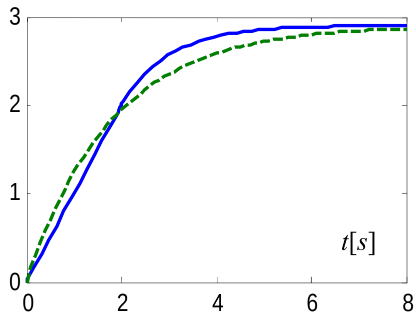


Fig. 5. Approximation of $H(s)$ by a first-order plant

Since $\alpha O \tau_t = G c_t \gg \rho c S$, we assume that it is possible to omit the time constant $\rho c S \tau_t / (\rho c S + \alpha O \tau_t)$ in equations (21) and (22). The transfer functions $F_q(s)$ and $F_\xi(s)$ then can be obtained in the simplified form

$$F_q(s) \approx \frac{K_q}{\tau_H s + 1}, \quad F_\xi(s) \approx -K_\xi \frac{\tau_t s + 1}{\tau_H s + 1} \tag{25}$$

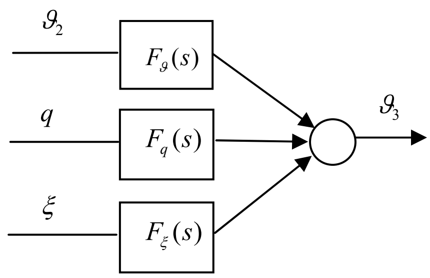


Fig. 6. Model of heating

where $K_q, K_\xi, \tau_t, \tau_H$ are positive constants. From Fig. 5 it is obvious that $\tau_H$ is not too much different from $T_d$. The model of heating is shown in Fig. 6.

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## REFERENCES

Čermák, J., Peterka, V., Závorka, J. (1968). *Dynamika regulovaných soustav v tepelné energetice a chemii*. Academia, Praha.

Karták, J., Janeba, B., Šula, O. (1981). *Dynamika a regulace parních kotlů*. SNTL, Praha.

Smith, C. A., Corripio, A. (2006). *Principles and Practice of Automatic Process Control*. Wiley, 3rd edition.

Goodwin, C. G., Payne, R. L. (1977). *Dynamic System Identification*. Academic Press, New York.

Ljung, L. (1999). *System Identification*. *Theory for the User*. Prentice Hall, 2nd edition.

Cvejn, J. (2009). Estimation of parameters of a model of a steam overheater. In. *Proc. 17th International Conference Process Control ’09*. Štrbské pleso, Slovakia.