Slovak University of Technology in Bratislava
Institute of Information Engineering, Automation, and Mathematics

PROCEEDINGS

17$^{\mathrm{th}}$ International Conference on Process Control 2009
Hotel Baník, Štrbské Pleso, Slovakia, June 9 – 12, 2009
ISBN 978-80-227-3081-5
http://www.kirp.chtf.stuba.sk/pc09

Editors: M. Fikar and M. Kvasnica

Cvejn, J.: Estimation of Parameters of a Model of a Steam Overheater from Service Data, Editors: Fikar, M., Kvasnica, M., In *Proceedings of the 17th International Conference on Process Control '09*, Štrbské Pleso, Slovakia, 174–178, 2009.

Full paper online: `http://www.kirp.chtf.stuba.sk/pc09/data/abstracts/014.html`

ESTIMATION OF PARAMETERS OF A MODEL OF A STEAM OVERHEATER FROM SERVICE DATA

J. Cvejn*

* *Univerzity of Pardubice, Faculty of Electrical Engineering and Informatics*
e-mail: jan.cvejn@upce.cz

Abstract: The paper describes a procedure of obtaining estimated values of parameters of a model of a controlled powerplant steam overheater from measured service data. A construction of a mathematical model of the overheater is described in a stand-alone paper in the same proceedings. The overheater consists of two subsystems that can be identified separately – cooling steam by water injection and heating steam part. Statistical estimation of parameters from measured data is complicated by the fact that the disturbances influencing the outlet temperature are not pure random. Moreover, since the plant is under feedback, correlation between the disturbances and the system input has to be considered.

Keywords: heat systems, mathematical modeling, system identification, control systems.

1 INTRODUCTION

In the paper [Cvejn 2009] published in the same proceedings we described a construction of a mathematical model of a controlled steam overheater of a powerplant boiler. The model is to be used subsequently for design or enhancement of control. We assume that unknown model parameters, or at least their approximate estimates, can be determined by processing measured service data. A scheme of the overheater is shown in Fig. 1.

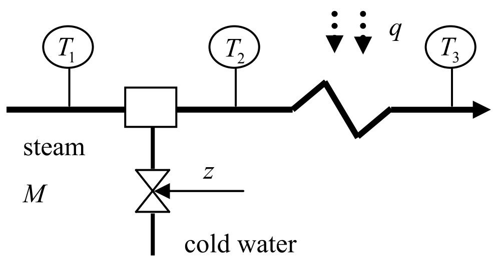

Fig. 1. Control of the temperature of overheated steam by cold water injection

Measured variables are the steam temperatures T_1, T_2, T_3, total steam flow rate M and the manipulated variable z, which controls cold water valve. The outlet temperature T_3 is regulated. Disturbances in operation are changes of the temperature T_1, the flow rate M and the heat input q. The last disturbance is not measurable.

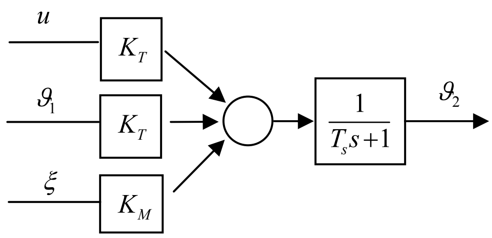

Fig. 2. Model of cooling by water injection

The system can be divided into two subsystems that can be identified separately: Cooling subsystem with input variables T_1, M, z and output T_2 and heating subsystem with input variables T_2, q and output T_3. For the first subsystem we obtained a linearized model in the form of transfer functions [Cvejn 2009]

$$F_z(s) = \frac{K_z}{T_s s+1},\ F_T(s) = \frac{K_T}{T_s s+1},\ F_M(s) = \frac{K_M}{T_s s+1} \qquad (1)$$

where K_z, K_T, K_M are the gains corresponding to changes of the control variable of the valve, the temperature of inlet steam and the flow and T_s is common cooling time constant (Fig. 2).

In Fig. 2, u, ϑ_1 and ξ are deviations of z, T_1 and M, respectively, from a chosen working point.

For the heating subsystem we obtained approximate transfer functions in the form [Cvejn 2009]:

$$F_\vartheta(s) = \left(\frac{\tau_1 s + 1}{\tau_2 s + 1}\right)^p e^{-T_d s} \quad (2)$$

$$F_q(s) = \frac{K_q}{\tau_H s + 1} \quad (3)$$

$$F_\xi(s) = -K_\xi \frac{\tau_t s + 1}{\tau_H s + 1} \quad (4)$$

where $K_q, K_\xi, \tau_1, \tau_2, \tau_t, \tau_H, T_d$ are positive constants and $p \geq 1$ is an integer parameter (Fig. 3). ϑ_3 and q are deviations of outlet temperature and the heat input.

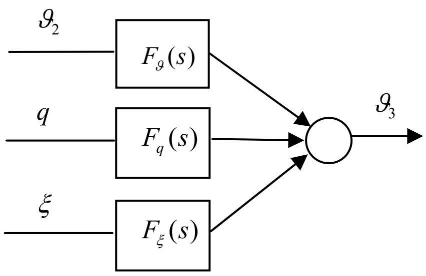

Fig. 3. Model of heating

The model parameters can be estimated by processing measured service data. Unfortunately, we were not able to realize special conditions advantageous for identification, such as steady state or special input signals. It is only possible to assume that in some time intervals the process variables and external influences fluctuate in a small neighborhood of their mean values. This requirement is also necessary for validity of the model.

2 CHOICE OF THE ESTIMATION METHOD

Under the conditions mentioned above the model parameters can be approximately determined using point estimation or correlation methods [Goodwin 1977], [Ljung 1999]. Since over large time intervals significant changes in the working state occur, we assume that it is not advantageous to process very large data files. Therefore, correlation methods are probably not preferable choice in this case, because they require large number of samples.

Statistical estimation of the model parameters is not trivial due to the following reasons:

- unmeasured disturbances influencing the system output are not pure random (due to the transfer function (3) the heat input disturbance sequence is correlated).

- the plant is under feedback (in fact, two feedbacks are in operation due to cascade configuration of controllers, see [Cvejn 2009], Fig. 2).

- the flow rate disturbance is measured, but its transfer function $F_\xi(s)$ denominator differs from the denominator of $F_\vartheta(s)$, so commonly used ARMAX model is not adequate.

Although precise configuration of the control system and its parameters are not known, the parameter estimation from measured data is still possible. We consider that the unmeasured disturbance, which corresponds to the heat input change, influencing the system output both directly and through the feedback, can be modeled as output of an unknown linear plant excited by Gaussian pure random noise. A higher quality estimate indeed could be obtained by considering known structure and parameters of the control system.

Assume that the process data can be modeled as

$$\mathbf{A}\mathbf{x} - \mathbf{b} = \boldsymbol{\varepsilon} \quad (5)$$

where the matrix $\mathbf{A}$ and the vector $\mathbf{b}$ are constructed from measured data, $\boldsymbol{\varepsilon}$ is the error vector and $\mathbf{x}$ is the vector of estimated parameters. We consider that the number of data is N_d and the number of equations in (5) is $N \leq N_d$. If the covariance matrix $\boldsymbol{\Sigma} = E\left\{\boldsymbol{\varepsilon}\boldsymbol{\varepsilon}^T\right\}$ had been known, a consistent estimate of $\mathbf{x}$, i.e. such that $\hat{\mathbf{x}} \to \mathbf{x}$ for $N \to \infty$, would be obtained as

$$\hat{\mathbf{x}} = \left(\mathbf{A}^T \boldsymbol{\Sigma}^{-1} \mathbf{A}\right)^{-1} \mathbf{A}^T \boldsymbol{\Sigma}^{-1} \mathbf{b} \quad (6)$$

[Goodwin 1977]. Although knowledge of $\boldsymbol{\Sigma}$ is not available, its estimate can be constructed using the estimate of $\mathbf{x}$. Repeating this procedure leads to iterative relaxation algorithm for determination of the estimates $\hat{\mathbf{x}}$ and $\hat{\boldsymbol{\Sigma}}$. Usually $\boldsymbol{\Sigma}$ is not determined directly, but the error sequence is parametrized using autoregressive model:

$$\varepsilon_k + c_1 \varepsilon_{k-1} + \ldots + c_n \varepsilon_{k-n} = w_k \quad (7)$$

where $\{w_k\}$ is the zero-mean Gaussian white noise sequence. From known sequence $\{\varepsilon_k\}$ the parameters $(c_1, \ldots, c_n)^T$ can be estimated using common least-squares method and consequently the disturbance impulse response $\{g_k\}$ can be constructed. ε_k can be expressed using convolution theorem in the form

$$\varepsilon_k = \sum_{i=0}^{\infty} g_i w_{k-i} \quad (8)$$

which can be after truncation rewritten into matrix form as $\boldsymbol{\varepsilon} = \mathbf{L}\mathbf{w}$, where

$$\mathbf{L} = \begin{pmatrix} 1 & & \\ g_1, 1 & 0 & \\ \ldots & & \\ g_{N-1}, & \ldots, & 1 \end{pmatrix}. \tag{9}$$

Now it is possible to express

$$\boldsymbol{\Sigma} = E\left\{\mathbf{L}\mathbf{w}\mathbf{w}^T\mathbf{L}^T\right\} = \mathbf{L}\mathbf{L}^T \tag{10}$$

and proceed by (6), but it is more advantageous to solve equivalent common least-squares problem

$$\left(\mathbf{L}^{-1}\mathbf{A}\right)\mathbf{x} - \left(\mathbf{L}^{-1}\mathbf{b}\right) = \mathbf{w} \tag{11}$$

where $\mathbf{L}^{-1}$ always exists due to special form of $\mathbf{L}$.

In this form the method is known as generalized least squares [Goodwin 1977]. Unfortunately, it is not true that the disturbance model (7) suits well, especially in the case of identification under feedback. Therefore, we tested a more general approach, where the error is not parameterized as in (7), but the components of the matrix $\boldsymbol{\Sigma}$ are estimated directly from the sequence $\{\varepsilon_k\}$ as

$$\Sigma_{i,i+j} = \Sigma_{i+j,i} = R_\varepsilon(j) \tag{12}$$

where $\{R_\varepsilon(j)\}$ is the autocovariance sequence of $\{\varepsilon_k\}$. After Cholesky decomposition into the product (10) the remaining procedure is the same (it does not matter that here the matrix $\mathbf{L}$ does not have units on diagonal in general). Unfortunately, for real data the decomposition (10) does not always exist and in such a case the relaxation algorithm fails. We tried to guarantee positive definiteness using a suitable correction of $\boldsymbol{\Sigma}$, but it seams that convergence of the relaxation algorithm is in this case rather exceptional.

Therefore, alternative methods were taken into account. Perhaps the most appealing approach in this case seems to be the maximum likelihood estimation [Goodwin 1977], [Ljung 1999]. We consider the error vector as a Gaussian zero-mean multivariable random variable with probability density function

$$p_{\Sigma}(\boldsymbol{\varepsilon}) = \frac{1}{\sqrt{(2\pi)^N \det\boldsymbol{\Sigma}}} \exp\left(-\frac{1}{2}\boldsymbol{\varepsilon}^T\boldsymbol{\Sigma}^{-1}\boldsymbol{\varepsilon}\right) \tag{13}$$

where $\boldsymbol{\Sigma} = \mathbf{L}\mathbf{L}^T$ and $\boldsymbol{\varepsilon}$ is determined from (5) given $\mathbf{x}$. The matrix $\mathbf{L}$ is constructed using (9) from a truncated impulse response $\{g_k\}$ stored in a vector $\mathbf{g}$. The function $p_{\Sigma(\mathbf{g})}(\boldsymbol{\varepsilon}(\mathbf{x}))$, which represents likelihood of the data for given $\mathbf{x}$ and $\mathbf{g}$, is maximized in the space of parameters $(\mathbf{x}, \mathbf{g})$. Negative logarithm of $p_{\Sigma}(\boldsymbol{\varepsilon})$ is

$$-\ln p_{\Sigma}(\boldsymbol{\varepsilon}) = \frac{N}{2}\ln(2\pi) + \frac{1}{2}\ln\det\boldsymbol{\Sigma} + \frac{1}{2}\boldsymbol{\varepsilon}^T\boldsymbol{\Sigma}^{-1}\boldsymbol{\varepsilon}. \tag{14}$$

Therefore, to obtain the maximum likelihood estimate of $(\mathbf{x}, \mathbf{g})$ the following function is numerically minimized:

$$\begin{aligned} J_N(\mathbf{x}, \mathbf{g}) &= \ln\det\left(\mathbf{L}(\mathbf{g})\mathbf{L}^T(\mathbf{g})\right) + \\ &+ \left(\mathbf{A}\mathbf{x} - \mathbf{b}\right)^T \mathbf{L}^{-T}(\mathbf{g})\mathbf{L}^{-1}(\mathbf{g})\left(\mathbf{A}\mathbf{x} - \mathbf{b}\right) = \\ &= \left\|\mathbf{L}^{-1}(\mathbf{g})\left(\mathbf{A}\mathbf{x} - \mathbf{b}\right)\right\|^2 \end{aligned} \tag{15}$$

because $\det\mathbf{L}(\mathbf{g}) = 1$. The minimization of (15) is a non-linear least squares problem that can be solved efficiently e.g. using Gauss-Newton method [Nocedal 1999].

3 ESTIMATION OF PARAMETERS OF THE COOLER SUBSYSTEM

The service data are sampled with period Δ. Discrete equivalents of cooling subsystem continuous transfer functions (1) are

$$\begin{aligned} &F_T(z) = K_T\frac{1-\alpha}{z-\alpha}, F_z(z) = K_z\frac{1-\alpha}{z-\alpha}, \\ &F_M(z) = K_M\frac{1-\alpha}{z-\alpha} \end{aligned} \tag{16}$$

where $\alpha = e^{-\Delta/T_s}$. If we choose the model of data in the form of difference equation

$$T_{2,k+1} - aT_{2,k} = bT_{1,k} + cz_k + dM_k + C + \varepsilon_k, \tag{17}$$

where $k = 1,\ldots,N-1$, C is a constant and ε_k is a zero-mean random error process, corresponding equation for deviations is

$$\vartheta_{2,k+1} - a\vartheta_{2,k} = b\vartheta_{1,k} + cu_k + d\xi_k + \varepsilon_k \tag{18}$$

and the continuous transfer functions parameters T_s, K_T, K_z, K_M can be easily obtained by comparison from estimated values a,b,c,d. Eq. (17) can be easily rewritten into the system (5), where $\mathbf{x} = (a,b,c,d,C)^T$ is the vector of estimated parameters and $N = N_d - 1$. Continuous transfer functions parameters are computed by comparison with (16).

Identification of this subsystem is rather straightforward. The values of the parameters obtained from different segments of data were similar enough, even when standard least-squares method was used.

4 ESTIMATION OF PARAMETERS OF THE HEATING PART

1. The case $p=1$

Z-transfer function corresponding to the continuous transfer function (2) for $p=1$ was obtained in the form

$$F_g(z)=\left(\frac{z}{z-1}-\frac{\beta z}{z-\alpha}\right)\frac{z-1}{z}z^{-d}= \\ =\frac{z(1-\beta)+(\beta-\alpha)}{z-\alpha}z^{-d} \tag{19}$$

where $\alpha=e^{-\Delta/\tau_2}$, $\beta=1-\tau_1/\tau_2$, Δ is the scan period and it is assumed that the time delay causes a shift of $d=T_d/\Delta$ steps.

If the flow rate influence is neglected, data of the plant (19) can be modeled as

$$T_{3,k+1}-a.T_{3,k}=b_1.T_{2,k+1-d}+b_0.T_{2,k-d}+C+\varepsilon_k \tag{20}$$

where *C* is a constant and ε_k is a zero-mean random error process. From the coefficients in the transfer function (19) it however follows that the unknown parameters are bound by the condition

$$a+b_1+b_0=1 \tag{21}$$

so one parameter can be eliminated and the vector of estimated parameters is $\mathbf{x}=(a,b_0,C)^T$ plus single unknown discrete parameter *d*. Parameter *d* was iteratively changed in a superior loop and was held constant in each estimation subproblem.

In the case of generalized least-squares estimate the optimal value of *d* was selected such that

$$\hat{\boldsymbol{\varepsilon}}^T\hat{\boldsymbol{\Sigma}}^{-1}\hat{\boldsymbol{\varepsilon}}\ /N\rightarrow\min_d . \tag{22}$$

Note that the number of equations depends on *d*, since $N=N_d-d-1$. The estimation of **x** is iterative itself. However, approximate value of $\hat{d}$ can be determined in a simplified way using common least squares estimator for determination of **x**.

In the case of the maximum likelihood estimate we select *d* so that $\min_{\mathbf{x},\mathbf{g}}\left\{J_N\left(\mathbf{x},\mathbf{g}\right)/N\right\}$ is minimal.

Practical results obtained so far are not very satisfactory. The values of coefficients obtained for different data segments differ in tens of percents. One reason may be too rough approximation of the transfer function $F_g(s)$. We believe that better results can be achieved by allowing $p>1$, as described below, but the implementation has not been finished yet.

2. The case $p>1$

The discrete transfer function is considered in the form

$$F_g(z)=\left(\frac{b_1z+b_0}{z-a}\right)^p z^{-d} \tag{23}$$

and corresponding discrete equation of *k*-th order can be easily determined. It is possible to estimate unknown coefficients of this equation using generalized least squares, but special structure of the transfer function (23) poses nonlinear constraints on their values.

Nevertheless, it seams that the solution can be obtained rather easily using maximum likelihood approach. In this case the unknown coefficients in the difference data model (5) are not considered as independent for optimization, but are determined as functions of α and β from (23) and (19). The objective function is again (15) and the remaining procedure remains the same.

We assume that the value *p* can be determined using knowledge of physical parameters of the overheater, see [Cvejn 2009, eq. 13], considering that *p* is close to κ.

3. Influence of the flow change disturbance

If the change of the flow rate is considered, history of ϑ_3 is described by the equation in *Z*-transform

$$\Theta_3(z)=\left(\frac{b_1z+b_0}{z-a}\right)^p z^{-d}\Theta_2(z)+\frac{d_1z+d_0}{z-c}\Xi(z). \tag{24}$$

Due to difference of denominators it is not possible to use linear regression directly. It is however possible to replace the transfer function $F_\xi(z)$ by the finite sum

$$F_\xi(z)\approx\frac{1}{(z-a)^p}\sum_{i=-p}^{m}c_{-i}z^{-i} \tag{25}$$

where c_i are unknown parameters and *m* is sufficiently large. In the case $p=1$ the corresponding model of data is

$$T_{3,k+1}-a.T_{3,k}=b_1.T_{2,k+1-d}+b_0.T_{2,k-d}+ \\ +\sum_{i=-1}^{m}c_{-i}M_{k-i}+C+\varepsilon_k \tag{26}$$

and the vector of unknown parameters is

$$\mathbf{x}=(a,b_0,c_1,c_0,...,c_{-m},C)^T . \tag{27}$$

Next procedure is analogous to the previous case. Unfortunately, it seems that in this case obtained impulse response corresponding to $F_\xi(z)$ contains oscillating components. Preventing from oscillations can be achieved by introducing constraints on c_i, but in this case a mathematical programming problem has to be solved. It was however observed that the flow change disturbance has minor effect and consequently it was neglected.

ACKNOWLEDGEMENTS

The work has been supported under the Czech research project MSM 0021627505. Thanks belong to management and technical personel of the powerplant International Power Opatovice, a.s., for providing measured service data and necessary technical information for the purposes of studying possibilities of enhancement of control of overheated steam temperature.

REFERENCES

Čermák, J., Peterka, V., Závorka, J. (1968). *Dynamika regulovaných soustav v tepelné energetice a chemii*. Academia, Praha.

Karták, J., Janeba, B., Šula, O. (1981). *Dynamika a regulace parních kotlů*. SNTL, Praha.

Smith, C. A., Corripio, A. (2006). *Principles and Practice of Automatic Process Control*. Wiley, 3rd edition.

Goodwin, C. G., Payne, R. L. (1977). *Dynamic System Identification*. Academic Press, New York.

Ljung, L. (1999). *System Identification*. *Theory for the User*. Prentice Hall, 2nd edition.

Nocedal, J., Wright, S. J. (1999). *Numerical Optimization*. Springer-Verlag.

Cvejn, J. (2009). Approximate Mathematical Model of a Steam Overheater. In. *Proc. 17th International Conference Process Control '09*. Štrbské pleso, Slovakia.