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MODEL MATCHING FOR NONLINEAR SYSTEMS NOT HAVING THE STATE-SPACE REALIZATION

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Abstract: In this technical note the model matching problem for nonlinear systems not having the state space realization is discussed. It is shown that even in such a case it is still possible to find a realizable compensator. To advantage, a transfer function formalism of nonlinear systems is employed.

Keywords: nonlinear systems, model matching, transfer functions, realizability

1. INTRODUCTION

In the model matching one studies the problem of designing a compensator for a control system under which the compensated system becomes transfer equivalent to a prespecified model. The model matching problem is thus a typical design problem and plays a key role in various other problems like the input-output linearization and the (disturbance) decoupling. In the linear case, one naturally requires the equality of the transfer functions of the model and of the compensated system. In the nonlinear case, the model matching was solved mainly within the state space approach by various authors, using problem statements which slighlty differ from one to the other (Benedetto and Isidori (1984); Benedetto (1990); Huijberts (1992); Conte et al. (2007)).
However, even if the Laplace transform of a nonlinear differential equation is not applicable, the transfer function formalism was recently developed also for nonlinear systems, see Zheng and Cao (1995); Halás and Huba (2006); Halás (2008); Halás and Kotta (2007ab); Halás (2007). Such a formalism generalizes well known results valid for linear time invariant systems and was already employed in Halás et al. (2008) to recast and solve the nonlinear model matching problem. It was shown that such an approach to the nonlinear model matching is more general, since neither the control system itself, nor the model and the compensator are required to be realizable in the state-space form. In particular, this gives a chance to find realizable compensators for nonlinear systems not having the state-space realization, forming the scope of our interest in this note.

2. NONREALIZABLE SYSTEMS

Control systems can be desribed in several ways. The most widely used are two following:

- higher order input-output differential equation,
- set of coupled first order differential equation (the so-called state space representation).

In the linear case, any control system desribed by a higher order input-output differential equation can be equivalently described by a set of coupled first order differential equation, which is the so-called state space representation, and vice versa.

However, this does not hold anymore when systems are nonlinear. Although for any state space representation of the form

$$\begin{aligned}\dot{x} &= f(x,u)\\ y &= g(x,u)\end{aligned} \tag{1}$$

a corresponding input-output differential equation

$$y^{(n)} = \varphi(y, \dot{y}, \ldots, y^{(n-1)}, u, \dot{u}, \ldots, u^{(m)}) \tag{2}$$

can be, at least locally, always found (Conte et al. (2007)), converse does not hold in general. There exists a class of input-output differential equations of the form (2) for which a state space representation of the form (1) simply does not exist. A typical example is given by the system

$$\ddot{y} = \dot{u}^2$$

for which we are simply not able to find any state-space description of the form (1). See Conte et al. (2007) for more details.

In such cases we, of course, cannot use state space approaches to the model matching. An alternative way to the solution of nonlinear model matching was recently outlined in Halás et al. (2008) employing the transfer function approach. In fact, a more general case was considered, since neither the control system itself, nor the model and the compensator are required to be realizable in the state-space form. In particular, this gives a chance to find realizable compensators for nonrealizable nonlinear systems.

3. TRANSFER FUNCTIONS OF NONLINEAR SYSTEMS

We start with the introduction of transfer function formalism of nonlinear systems, following the lines of Halás and Huba (2006); Halás (2008); Halás et al. (2008).

Consider the SISO nonlinear system defined by an input-output equation of the form (2) where φ is assumed to be an element of the field of meromorphic functions $\mathcal{K}$.

The left skew polynomial ring $\mathcal{K}[s]$ of polynomials in s over $\mathcal{K}$ with the usual addition, and the (non-commutative) multiplication given by the commutation rule

$$sa = as + \dot{a} \tag{3}$$

where $a \in \mathcal{K}$, represents the ring of linear ordinary differential operators that act over vector space of one-forms $\mathcal{E} = \mathrm{span}_{\mathcal{K}}\{\mathrm{d}\xi; \xi \in \mathcal{K}\}$ in the following way

$$\left(\sum_{i=0}^{k} a_i s^i\right) v = \sum_{i=0}^{k} a_i v^{(i)}$$

for any $v \in \mathcal{E}$.

The commutation rule (3) actually represents the rule for differentiating.

Lemma 1. (Ore condition). For all non-zero $a, b \in \mathcal{K}[s]$, there exist non-zero $a_1, b_1 \in \mathcal{K}[s]$ such that $a_1 b = b_1 a$.

Thus, the ring $\mathcal{K}[s]$ can be embedded to the non-commutative quotient field $\mathcal{K}\langle s\rangle$ by defining quotients as

$$\frac{a}{b} = b^{-1} \cdot a$$

The addition and multiplication in $\mathcal{K}\langle s\rangle$ are defined as

$$\frac{a_1}{b_1} + \frac{a_2}{b_2} = \frac{\beta_2 a_1 + \beta_1 a_2}{\beta_2 b_1}$$

where $\beta_2 b_1 = \beta_1 b_2$ by Ore condition and

$$\frac{a_1}{b_1} \cdot \frac{a_2}{b_2} = \frac{\alpha_1 a_2}{\beta_2 b_1} \tag{4}$$

where $\beta_2 a_1 = \alpha_1 b_2$ again by Ore condition.

Due to the non-commutative multiplication (3) they, of course, differ from the usual rules. In particular, in case of the multiplication (4) we, in general, cannot simply multiply numerators and denominators, nor cancel them in a usual manner. We neither can commute them as the multiplication in $\mathcal{K}\langle s\rangle$ is non-commutative as well.

Once the fraction of two skew polynomials is defined we can introduce the transfer function of the nonlinear system (2) as an element $F(s) \in \mathcal{K}\langle s\rangle$ such that $\mathrm{d}y = F(s)\mathrm{d}u$.

After differentiating (2) we get

$$\mathrm{d}y^{(n)} - \sum_{i=1}^{n-1} \frac{\partial\varphi}{\partial y^{(i)}} \mathrm{d}y^{(i)} = \sum_{i=0}^{m} \frac{\partial\varphi}{\partial u^{(i)}} \mathrm{d}u^{(i)}$$

or alternatively

$$a(s)\mathrm{d}y = b(s)\mathrm{d}u$$

where $a(s) = s^n - \sum_{i=1}^{n-1} \frac{\partial\varphi}{\partial y^{(i)}} s^i$ and $b(s) = \sum_{i=0}^{m} \frac{\partial\varphi}{\partial u^{(i)}} s^i$ are in $\mathcal{K}[s]$. Then

$$F(s) = \frac{b(s)}{a(s)}$$

Example 2. Consider the system

$$\ddot{y} = -y + \dot{u}^2 + u$$

After differentiating

$$\begin{aligned}\mathrm{d}\ddot{y} &= -\mathrm{d}y + 2\dot{u}\mathrm{d}\dot{u} + \mathrm{d}u\\ (s^2+1)\mathrm{d}y &= (2\dot{u}s + 1)\mathrm{d}u\end{aligned}$$

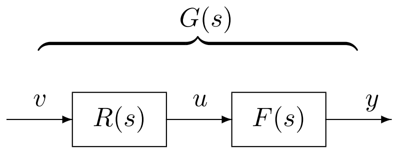

Fig. 1. Compensated system

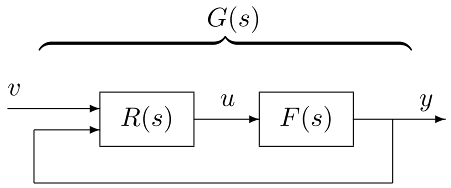

Fig. 2. Compensated system

and the transfer function is

$$F(s) = \frac{2\dot{u}s + 1}{s^2 + 1}$$

Transfer functions of nonlinear systems satisfy many properties we expect from transfer functions (Halás (2008)). Here, the most important is that we can use transfer function algebra when combining systems in series, parallel or feedback connection and that two nonlinear systems are locally transfer equivalent (admit the same irreducible input-output differential equation) if and only if they have the same transfer function (Perdon et al. (2007)).

4. MODEL MATCHING

Thus, in the nonlinear model matching one, as in linear case, requires the equality of the transfer functions of the model and that of the compensated system. This was recently discussed in Halás et al. (2008) where introduced transfer function formalism was employed to recast and solve the model matching problem of single-input single-output nonlinear control systems. This resulted in designing compensators, both feedforward, Fig. 1, and feedback, Fig. 2, under which the input-output map of the compensated system becomes transfer equivalent to a prespecified model $G(s)$. It was shown that the existence of a feedforward compensator requires a restrictive integrability condition, while a feedback compensator exists whenever the system is nontrivial, that is $F(s) \neq 0$. See Halás et al. (2008) for technicalities.

4.1 *Model matching problem for nonrealizable systems*

The input-output approach to the model matching problem, as presented in Halás et al. (2008), is applicable also to nonlinear systems not having the state-space realization. We do not require this from the original system equations neither from compensator equations. So there is a chance to find realizable compensators for nonrealizable systems in both feedforward and feedback case.

Example 3. Consider the system from Example 2 which has, according to Conte et al. (2007), no state-space realization of the form (1). The transfer function was

$$F(s) = \frac{2\dot{u}s + 1}{s^2 + 1}$$

Now, let the desired dynamics of the compensated system be given by the transfer function

$$G(s) = \frac{1}{s^2 + 1}$$

To find a feedforward compensator (Halás et al. (2008)) depicted in Fig. 1 we compute

$$R(s) = F^{-1}(s) \cdot G(s) = \frac{1}{2\dot{u}s + 1} \tag{5}$$

The compensator's equation is integrable

$$\begin{aligned} (2\dot{u}s + 1)\mathrm{d}u &= \mathrm{d}v \\ 2\dot{u}\mathrm{d}\dot{u} + \mathrm{d}u &= \mathrm{d}v \\ \dot{u}^2 + u &= v \end{aligned}$$

and has the following state-space realization

$$\begin{aligned} \dot{\xi} &= \sqrt{v - \xi} \\ u &= \xi \end{aligned}$$

Note that for the system considered here the feedback compensator depicted in Fig. 2 results in the same compensator $R(s)$ as in the feedforward case.

4.2 *Realizability of compensators*

To pass out the model matching problem it is, of course, necessary that the designed compensator is realizable itself, otherwise it cannot be implemented. This can easily happen.
In dealing with this problem we can follow the lines of Halás and Kotta (2009 submitted) where the realizability problem of nonlinear system is stated and solved within the transfer function formalism. In particular, the notion of the so-called adjoint polynomials play a key role.

Example 4. Consider the compensator (5) from Example 3 with the transfer function

$$R(s) = \frac{1}{2\dot{u}s + 1} = \frac{\frac{1}{2\dot{u}}}{s + \frac{1}{2\dot{u}}}$$

Using the notion of adjoint polynomials we get its adjoint transfer function as

$$R^*(s^*) = \frac{\frac{1}{2\ddot{u}}}{s^* + \frac{1}{2\ddot{u}}}$$

and the compensator has a realization of the form (1) if and only if the one-form

$$\frac{1}{2\ddot{u}}\mathrm{d}v - \frac{1}{2\ddot{u}}\mathrm{d}u$$

is integrable. Note that $\dot{u} = \sqrt{v-u}$ and thus

$$\frac{1}{2\sqrt{v-u}}\mathrm{d}v - \frac{1}{2\sqrt{v-u}}\mathrm{d}u = \mathrm{d}\sqrt{v-u}$$

which means that the compensator is realizable. Finally, choosing $\xi = u$ yields the same state space realization as in Example 3.

5. CONCLUSIONS

In this note we discussed the model matching problem for nonlinear systems not having the state space realization. It was shown that even in such a case it is possible to find a realizable compensator. To find a solution, we, to advantage, employed transfer functions of nonlinear systems.

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