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THE FIRST RESULTS OF SYSTEMS IDENTIFICATION METHODS FOR FMRI DATA

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Abstract: The main goal of this paper is using identification methods as a certain alternative to Dynamic Casual Modeling (DCM) analysis which detects the so-called intrinsic connections among selected brain areas. In recent years it has been shown that the similar problems, as there appear in functional Magnetic Resonance Imaging(fMRI) area, are formulated in dynamic system identification and estimation tasks. The subspace identification methods were chosen for this identification procedure because they prove good results for Multiple Input Multiple Output (MIMO) systems identification. These methods produce state space description of identified system [2].
The main part of this paper deals with the quality of identification results depending on some important data parameters. Consequently the processing of final state space description into more suitable form follows.

Keywords: Subspace identification methods, MIMO system, DCM analysis.

1. INTRODUCTION

The dynamic system identification methods are usually used for control engineering. The task of DCM procedure is markedly similar as the task of identification of MIMO (Multiple Input Multiple Output) systems. There offers an idea to use some suitable identification methods as a certain alternative to DCM procedure. If the dynamic identification methods are successful, some drawbacks of DCM procedure will be eliminated. There might appear some complications with the structure of the whole system. The basic idea is to separate the system dynamics into two parts, illustrated in the Fig.1. The subspace identification methods have good results for identification of MIMO systems. So we decided to use them as potential alternative to DCM procedure.

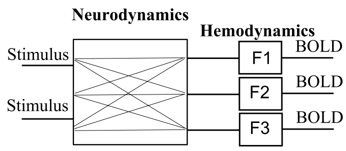

Fig. 1. The principal diagram of identification procedure

2. DCM PROCEDURE

Dynamic Casual Modeling (DCM) is a statistical technique for detection of connections among selected brain areas [4]. The DCM procedure treats the brain as a deterministic nonlinear dynamic system. This system has some inputs and produces some outputs, measured by fMRI as the BOLD (Blood Oxygen Level Dependent) signals. The inputs to the system are the signals defining a certain fMRI experiment, i.e. time series representing some stimuli such as finger movement commands, projection of emotional pictures to the patients etc. Furthermore a model structure must be predefined before the DCM analysis is applied. Therefore certain special knowledge in brain organization is necessary. In addition, there are typically several structure candidates that must be processed and evaluated separately which can become time consuming.
The DCM models are estimated using Bayesian estimators [5]. The inferences about connections are made using the posterior or conditional density [4]. The DCM result is the likeliest model accompanied by strength values of significant connections.

3. SUBSPACE IDENTIFICATION METHODS

Subspace identification methods combine results of systems theory, geometry and numerical linear algebra [2]. They can be thought of as modern capable alternative to ARX, ARMAX, OEM and other classical procedures for fitting linear dynamical models to measured data. Their benefits come up especially in the case of MIMO systems (with many inputs and/or outputs). The main advantage is a small number of parameters defined by users. Basically it is only a system order. These identification methods even offer the estimation of that.

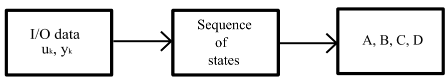

Fig. 2. The basic principle of subspace identification method

The identification algorithm has two principal steps [2]. First, a projection of certain subspaces generated from the data is calculated. The important term is Hankel matrix which is characterized by constant skew-diagonals. It contains past and future I/O data. The intersection row subspaces between past and future data produces the row subspace of states. An estimate can also be found of the extended observability matrix and an estimate of the states of the unknown system is given. The second step of the algorithm uses the extended observability matrix or the estimated states for retrieving the state space matrices for instance by means of least squares.

4. PRACTICAL EXAMPLES

This section deals with practical examples associated with identification task by means of subspace methods and DCM procedure. The several cases discuss the accuracy of identification results depending on important data parameters. The typical fMRI data suitable for DCM procedure is generated by Statistical Parametric Mapping (SPM) simulator. For subspace identification procedure is used Identification Matlab toolbox.

4.1 *Identification - examples*

This section summarizes the first results of subspace identification experiments for fMRI simulated data. The simulated data sets differ in the signal-to-noise ratio factor (SNR) and in number of samples (scans). Other parameters are the number of regions, sample period (TR), and the number of conditions (inp.), see [1] for details. The data parameters are presented in the tables 1 for particular cases. Related tables show the vector of onsets and vector of duration (definition of inputs). The last piece of information for the SPM simulator is the matrix *A* defining the strength of connections, and the input matrix *C*. The results of identification for different data sets are the identified matrix *A* acquired from SPM toolbox by DCM estimation [1], and then the (linear dynamic) model of simulated data acquired from the Identification Toolbox by help of subspace identification method [3].

4.1.1. *Case 1*

SNR	reg.	TR	scans	inp.		
50	3	1.7	256	1		
ons.	20	45	113	154	203	240
dur.	3	4	3	3	2	3

Table 1. The simulated data parameters - case 1,2

This example tests the quality of identification for simulated data with ”good” parameters. The data set has enough samples and the signal-to-noise ratio is high. See table 1. The input data is defined by vectors and the matrix of connection strength as well as the input matrix are also presented in Eq.1.

$$A = \begin{bmatrix} -1 & 0 & 0 \\ 1 & -1 & 1 \\ 2 & 0 & -1 \end{bmatrix} \qquad C = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \tag{1}$$

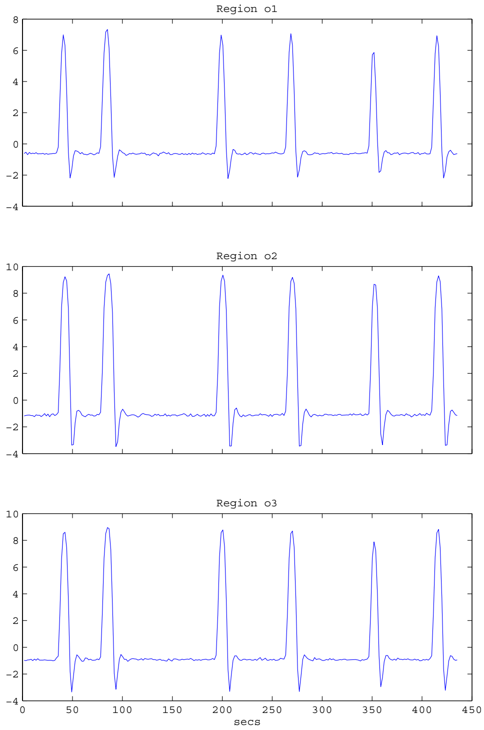

Fig. 3. The simulated data for three regions - case 1

The DCM procedure gives fairly good results in terms of the identified matrix A, see Eq.2 which corresponds to the simulation model's A Eq.1. The identification toolbox also proves useful here and fits successfully the simulated data by the identified linear model of order five, see Fig.4.

$$A_{DCM} = \begin{bmatrix} -1 & 0 & 0 \\ 0.872 & -1 & 1.0716 \\ 1.9955 & 0 & -1 \end{bmatrix} \tag{2}$$

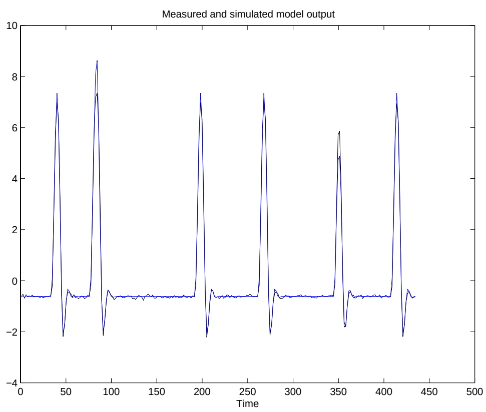

Fig. 4. The model of simulated data - case 1

4.1.2. *Case 2*

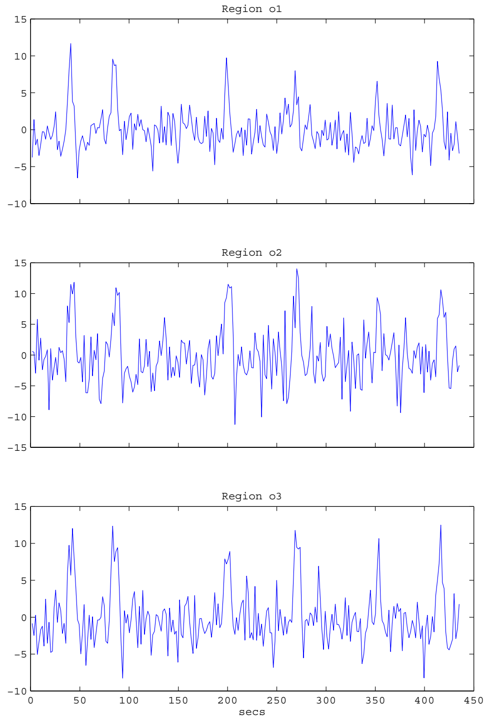

Fig. 5. The simulated data for three regions - case 2

Simulated data with smaller signal-to-noise ratio equal to one are processed now. Other parameters remain unchanged from the previous case. To compare the noise effect for the cases 1 and 2 see Fig.3 and Fig.5. The DCM procedure naturally embodies worse results than in the previous case which is shown in the matrix A Eq.3 again. The system identification toolbox identifies the model with order three and the identified output series is confronted with simulated data in the Fig.6.

$$A_{DCM} = \begin{bmatrix} -1 & 0 & 0 \\ 0.5057 & -1 & 0.5106 \\ 0.9121 & 0 & -1 \end{bmatrix} \tag{3}$$

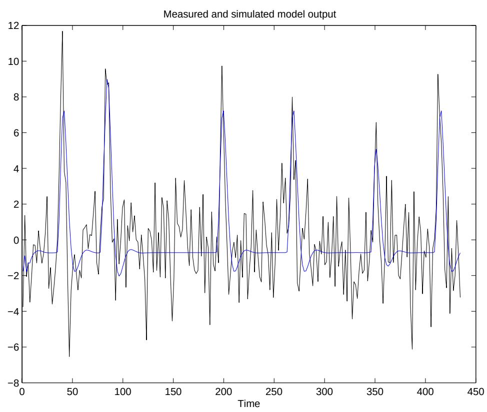

Fig. 6. The model of simulated data - case 2

4.1.3. *Case 3* A reduced 64-samples set was generated by SPM simulator. The signal-to-noise ratio is the same as in the case 1 ($= 50$). The vectors of onsets and durations differ see table 2. The connectivity matrix is copied from the previous cases. DCM result matrix A is Eq.4.

SNR	reg.	TR	scans	inp.		
50	3	1.7	64	1		
ons.	5	12	29	39	51	60
dur.	2	2	3	1	2	2

Table 2. The simulated data parameters - case 3,4

$$A_{DCM} = \begin{bmatrix} -1 & 0 & 0 \\ 0.7566 & -1 & 1.1719 \\ 1.91 & 0 & -1 \end{bmatrix} \tag{4}$$

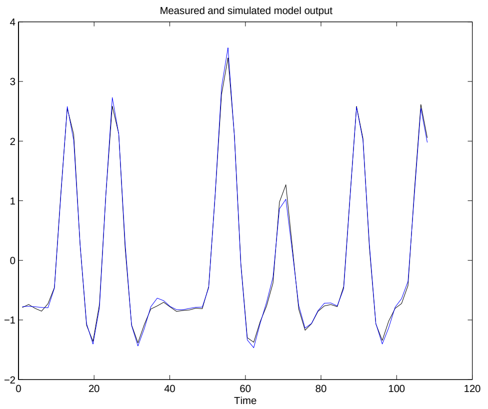

Fig. 7. The model of simulated data - case 3

4.1.4. *Case 4* DCM result connectivity matrix is

$$A_{DCM} = \begin{bmatrix} -1 & 0 & 0 \\ 0.7294 & -1 & 0.3973 \\ 0.6828 & 0 & -1 \end{bmatrix} \tag{5}$$

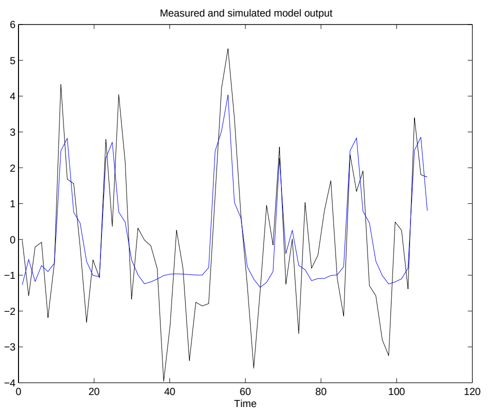

Fig. 8. The model of simulated data - case 4

The combination of small signal-to-noise ratio and small number of samples is poor and identification failed seeing Fig.8.

5. BRAIN SYSTEM - SPECIAL STRUCTURE

From the previous section it is obvious that the subspace identification methods could be suitable for identification procedure with fMRI data and we can use them as a certain alternative to DCM procedure. Let's deal with the next step connected with transformation of state space description into more suitable form.

The result of subspace identification methods is the state space description in form matrices A, B, C and D. Matrix A represents the dynamics, B is related to inputs and C includes outputs. There might appear an ambiguity in marking of matrices because DCM procedure uses matrix B for representing modulatory inputs and matrix C for external inputs. The modulatory inputs influence the connections among brain regions directly while the external inputs influence the regions. We decided not to consider modulatory inputs and used marking typical for state space description related to process control. Indeed it means to consider matrix A for dynamics, B for inputs and C for outputs as was noticed above.

The structure of whole system was shown in one of the previous pictures Fig.1. If we look at this picture we can see the structure of identified system with two main parts called neurodynamics and hemodynamics. Each brain region has own dynamics (hemodynamics) comprised by filter whose output is measured signal called BOLD signal. The first part called neurodynamics represents the intrinsic connections among brain regions. If we want to detect these connections we will have to transform the final full matrices A, B, C and D into the form according to this structure shown at the Fig.1.

If we describe the structure of whole system in detail we can say that the matrix A contains the eigenvalues of output filters representing hemodynamics, then their gain coefficients and finally the submatrix which defines the intrinsic connections among brain regions. Matrix B represents the structure of inputs and matrix C structure of outputs. Matrix D is zero therefore we won't pay attention to this matrix. In the following section there are practical examples related to this transformation.

5.1 *Example - two regions, the first order filters*

This case deals with identification for two brain regions, one input signal and two output signals. This system could represent simple brain system with two regions, the structure of final matrices is according to the structure defined by Fig.9. The first step is creating of the suitable system matrices called final matrices Eq.6 7 8 9 10. Then the identification procedure follows. The result of the subspace identification methods is state space description in the form of full matrices A, B, C and D. Matrix D is zero in both cases therefore it is not noted here. So the next part is looking for transformation steps which achieve to transform the full matrices to final matrices.

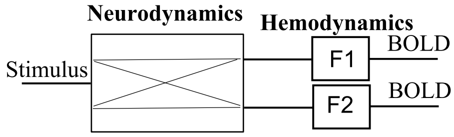

Fig. 9. The system structure

$$A_{full} = \begin{array}{cc} & \begin{array}{cccc} x_1 & x_2 & x_3 & x_4 \end{array} \\ \begin{array}{c} x_1 \\ x_2 \\ x_3 \\ x_4 \end{array} & \begin{bmatrix} -1.658 & 1.990 & -5.565 & 12.270 \\ -6.095 & -5.173 & -0.386 & -5.901 \\ 2.803 & 4.866 & -0.678 & 1.988 \\ -2.198 & -12.690 & 5.117 & -15.490 \end{bmatrix} \end{array} \tag{6}$$

$$A_{final} = \begin{array}{cc} & \begin{array}{cccc} x_1 & x_2 & x_3 & x_4 \end{array} \\ \begin{array}{c} x_1 \\ x_2 \\ x_3 \\ x_4 \end{array} & \begin{bmatrix} -1 & 0 & 1 & 0 \\ 0 & -2 & 0 & 2 \\ 0 & 0 & -10 & 0 \\ 0 & 0 & 5 & -10 \end{bmatrix} \end{array} \tag{7}$$

$$B_{full} = \begin{array}{cc} & u_1 \\ \begin{array}{c} x_1 \\ x_2 \\ x_3 \\ x_4 \end{array} & \begin{bmatrix} 3.084 \\ -1.100 \\ 0.665 \\ -3.493 \end{bmatrix} \end{array} \qquad B_{final} = \begin{array}{cc} & u_1 \\ \begin{array}{c} x_1 \\ x_2 \\ x_3 \\ x_4 \end{array} & \begin{bmatrix} 0 \\ 0 \\ 1 \\ 1 \end{bmatrix} \end{array} \tag{8}$$

$$C_{full} = \begin{array}{cc} & \begin{array}{cccc} x_1 & x_2 & x_3 & x_4 \end{array} \\ \begin{array}{c} y_1 \\ y_2 \end{array} & \begin{bmatrix} -1.046 & 0.683 & 0.228 & -1.096 \\ -1.713 & 0.764 & -0.150 & -1.702 \end{bmatrix} \end{array} \tag{9}$$

$$C_{final} = \begin{array}{cc} & \begin{array}{cccc} x_1 & x_2 & x_3 & x_4 \end{array} \\ \begin{array}{c} y_1 \\ y_2 \end{array} & \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \end{array} \tag{10}$$

Right now I would like to describe the steps of similarity transformations which are needed for creating required form of identified system. Each step of whole transformation abides by following rules Eq.14. Each transformation step is connected with previous step and the transformation matrix is usually made from final matrices of previous step.

$$A_{new} = T^{-1}AT \tag{11}$$

$$B_{new} = T^{-1}B \tag{12}$$

$$C_{new} = CT \tag{13}$$

$$D_{new} = D \tag{14}$$

At first the eigenvalues are obtained by means of Schur decomposition. This transformation enables to acquire the zero elements under the main diagonal containing these eigenvalues. Through the function called *ordschur* we can arrange the eigenvalues on basis of our requirements. The following transformation makes the diagonal submatrix appropriate to filters' dynamics (hemodynamics). This transformation matrix contains the eigenvectors of relevant submatrix (related to filters' dynamics). The results of this step follow 15 16 17.

$$A_{step1} = \begin{array}{cc} & \begin{array}{cccc} x_1 & x_2 & x_3 & x_4 \end{array} \\ \begin{array}{c} x_1 \\ x_2 \\ x_3 \\ x_4 \end{array} & \begin{bmatrix} -1 & 0 & -28.030 & 26.640 \\ 0 & -2 & 36.250 & -35.150 \\ 0 & 0 & -10 & 19.780 \\ 0 & 0 & 0 & -10 \end{bmatrix} \end{array} \tag{15}$$

$$B_{step1} = \begin{array}{cc} & u_1 \\ \begin{array}{c} x_1 \\ x_2 \\ x_3 \\ x_4 \end{array} & \begin{bmatrix} -6.573 \\ 8.751 \\ -3.566 \\ 1.269 \end{bmatrix} \end{array} \tag{16}$$

$$C_{step1} = \begin{array}{cc} & \begin{array}{cccc} x_1 & x_2 & x_3 & x_4 \end{array} \\ \begin{array}{c} y_1 \\ y_2 \end{array} & \begin{bmatrix} -0.279 & 0 & 0.869 & 0.998 \\ 0 & 0.329 & 1.440 & 1.180 \end{bmatrix} \end{array} \tag{17}$$

The next step is adjustment of matrix C especially elements appropriate to states which don't participate in output of whole system. The main part of transformation matrix is null space of matrix C_{step1}. So the result is Eq.18 19 20.

$$A_{step2} = \begin{array}{cc} & \begin{array}{cccc} x_1 & x_2 & x_3 & x_4 \end{array} \\ \begin{array}{c} x_1 \\ x_2 \\ x_3 \\ x_4 \end{array} & \begin{bmatrix} -1 & 0 & 1.280 & -1.270 \\ 0 & -2 & -2.919 & 3.207 \\ 0 & 0 & -44.500 & 34.220 \\ 0 & 0 & -34.790 & 24.500 \end{bmatrix} \end{array} \tag{18}$$

$$B_{step2} = \begin{array}{cc} & u_1 \\ \begin{array}{c} x_1 \\ x_2 \\ x_3 \\ x_4 \end{array} & \begin{bmatrix} 0.001 \\ -0.001 \\ -9.446 \\ -6.701 \end{bmatrix} \end{array} \tag{19}$$

$$C_{step2} = \begin{array}{c|cccc} & x_1 & x_2 & x_3 & x_4 \\ \hline y_1 & -0.279 & 0 & 0 & 0 \\ y_2 & 0 & 0.329 & 0 & 0 \end{array} \qquad (20)$$

The matrix A contains submatrix which includes the gain coefficients of output filters. The next step transforms this submatrix according to Fig.9 by means of inverse matrix.

$$A_{step3} = \begin{array}{c|cccc} & x_1 & x_2 & x_3 & x_4 \\ \hline x_1 & -1 & 0 & 1 & 0 \\ x_2 & 0 & -2 & 0 & 1 \\ x_3 & 0 & 0 & -10 & 0 \\ x_4 & 0 & 0 & -8.486 & -10 \end{array} \qquad (21)$$

$$B_{step3} = \begin{array}{c|c} & u_1 \\ \hline x_1 & 0.001 \\ x_2 & -0.001 \\ x_3 & -3.585 \\ x_4 & 6.080 \end{array} \qquad (22)$$

$$C_{step3} = \begin{array}{c|cccc} & x_1 & x_2 & x_3 & x_4 \\ \hline y_1 & -0.279 & 0 & 0 & 0 \\ y_2 & 0 & 0.329 & 0 & 0 \end{array} \qquad (23)$$

Matrix B is changed on basis of knowledge of output filters. The filters are defined by their eigenvalues and gain coefficients in matrix C, see below Eq.24 25 26.

$$A_{step4} = \begin{array}{c|cccc} & x_1 & x_2 & x_3 & x_4 \\ \hline x_1 & -1 & 0 & -0.279 & 0 \\ x_2 & 0 & -2 & 0 & 0.329 \\ x_3 & 0 & 0 & -10 & 0 \\ x_4 & 0 & 0 & -8.486 & -10 \end{array} \qquad (24)$$

$$B_{step4} = \begin{array}{c|c} & u_1 \\ \hline x_1 & 0 \\ x_2 & 0 \\ x_3 & -3.585 \\ x_4 & 6.080 \end{array} \qquad (25)$$

$$C_{step4} = \begin{array}{c|cccc} & x_1 & x_2 & x_3 & x_4 \\ \hline y_1 & 1 & 0 & 0 & 0 \\ y_2 & 0 & 1 & 0 & 0 \end{array} \qquad (26)$$

Then only adjustment of gain coefficients follows, especially in matrices A and B. This step transform only individual state but it is always adjusted in all matrices.

6. CONCLUSION

The main goal of this paper is using dynamic system identification methods for modeling fMRI data. The important aim is to develop certain alternative to DCM procedure (for detection of connections among brain regions) by help of subspace identification methods which would eliminate some drawbacks of DCM procedure. The subspace identification methods are successful for identification of ”brain” system. However, the similarity transformation is more complicated for that. The first problem is variable number of regions. And the order of output filters must be at least the second. It means that matrices of state space description will be larger. Unfortunately the mentioned steps of transformation are not general. So we can try to adjust the subspace identification methods. It would mean to enforce the specific structure to subspace identification methods during identification procedure.

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