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MODELING AND CONTROL OF THERMAL PLANT

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Abstract: The aim of this paper is to analyze the efficiency of raising the order of the linear model in modeling thermal plant. Results achieved both in identification and control are compared by considering plant models with the dominant dynamics of the 1st, 2nd and 3rd order. For each model the most appropriate controller is evaluated by simulation and real time control. The simulations and the real experiment results are compared to determine which model is more appropriate to describe the plants dynamics.

Keywords: linear model, thermal plant, pole assignment control, saturation.

1 INTRODUCTION

Control of thermal plant is frequently used to demonstrate properties of control design approaches [8], [9], [10], [12], [20], [21], [22], [27]. Thereby, different simplifications are considered, as e.g. approximation of the plant dynamics by the 1st order transfer function [21]. Of course, the question arises, how far such simplifications influence the resulting control quality and how far this can be improved by considering more complex model.

2 THERMO-OPTICAL PLANT

The thermo-optical plant laboratory model (Fig.1) offers measurement of 8 process variables: controlled temperature, its filtered value, ambient temperature, controlled light intensity, its derivative and filtered value, the fan speed of rotation and current. The temperature and the light intensity control channels are interconnected by 3 manipulated voltage variables influencing the bulb (heat & light source), the light-diode (the light source) and the fan (the system cooling). Besides these, it is possible to adjust two parameters of the light intensity derivator. Within Matlab/Simulink or Scilab/Scicos schemes [10] the plant is represented as a single block and so limiting needs on costly and complicated software packages for real time control. The (supported) external converter cards are necessary just for sampling periods below 50ms. Currently, more than 40 such plants are used in labs of several EU universities.

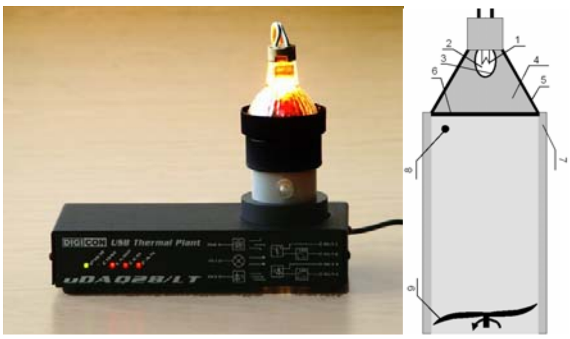

Fig. 1. The thermo-optical plant and scheme of its thermal channel

The thermal plant consists of a halogen bulb 12V DC/20W (elements 1-6), of a plastic pipe wall (element 7), of its internal air column (element 8) containing the temperature sensor PT100, and of a fan 12V DC/0,6W (element 9 that can be used for producing disturbances, but also for control). Next, we will consider temperature control by the bulb voltage. In paper [12] the plant dominant dynamics was analytically described by the 2nd order nonlinear model. However, it is known that heat is usually transferred by three different modes – conduction, convection and radiation. In this paper we will experimentally analyze this physical problem from the control point of view.

3 CONTROL BASED ON 1ST ORDER DOMINANT DYNAMICS

First order model + time constant

In the simplest case, when it is considered the heat transfer just with single mode, i. e. the plant is described by the dominant first order transfer function (differential equation). The time constant T_d is used for approximating non-modelled dynamics. Identification by genetic algorithms yields

$$G_{1N}(s) = \frac{K_1}{T_1 s + 1} \frac{1}{T_d s + 1} \tag{1}$$

$T_1 = 505.9055$ s; $K_1 = 7.8194$; $T_d = 20$ s

PI_1-controller

Let us consider a piecewise constant reference signal $w(t)$, the controlled output variable y , the control (manipulated) variable $u(t)$ and the required closed loop dynamics

$$F_w(s) = Y(s)/W(s) = -\alpha/(s-\alpha) \tag{2}$$

characterized by the closed loop pole $\alpha < 0$. In the presence of input disturbances v the required dynamics (2) can be achieved by the P-controller

$$\begin{gathered} u_r = sat\{K_R e + u_w - v\}; \\ e = w - y;\ K_R = -\frac{\alpha T_1 + 1}{K_1};u_w = \frac{w}{K_1} \end{gathered} \tag{3}$$

In order to get monotonic transients without overshooting in the presence of the non-modelled dynamics approximated by the time constant T_d, the closed loop pole should be restricted to the interval

$$\alpha \in (-1/(4T_d),\ 0) \tag{4}$$

whereby the limit admissible pole value

$$\alpha_e = -1/(4T_d) \tag{5}$$

corresponds to the double real dominant pole of the closed loop system with the P-controller (3) and the plant (1).

The disturbance observer (DOB) based I action can be introduced by reconstructing the plant input disturbance v by means of an inverse plant model as

$$v = \frac{1}{K_1}\frac{1+T_1 s}{1+T_f s}Y(s) - \frac{1}{1+T_f s}U_r(s) \tag{6}$$

This value is then used in (3).

By the analytical derivation of the controller structure, the DOB based approach is very close to the IMC one [23]. However, it does not use the parallel plant model that is appropriate for output disturbances, but the filtered inverse plant model focusing on input disturbances.

In motion control the DOB concept was introduced and applied in different modifications by Ohnishi and co-workers [12]. Umeno and Hori [25] developed the DOB theory by the factorization approach for the general first order system. DOB that is now used in many high precision motion control systems [17], [27] enables to improve the traditional trade-off between the stability and the response dynamics [13]. Due to the transparent relationship between performance criteria and gain selection, the DOB structure allows simple and intuitive tuning of its gains that is practically independent of the state feedback gains. This explains why DOB is so welcome by control practitioners. The only known limitation of the DOB concept is that it cannot be directly applied to the systems with a non-minimum phase zero. Similar consequences may also be expected when DOB based controllers are applied to control heat transfer, where the traditional PI controllers were applied by default. The result should be improved trade-off between the stability and the response dynamics, loop simplicity enabling simple and intuitive plant identification, loop tuning and consideration of constraints put on the control signal.

4 CONTROL BASED ON 2ND ORDER DOMINANT DYNAMICS

When considering the nominal dynamics of the plant with two different mode – the slow and fast one [6], [8], and [18], it could correspond to thermal plant with two ways of heat transfer [3], [16] – e.g. with the heat radiation (fast mode) and the heat conduction via body of the plant (slow mode). Such a situation could be characterized by the plant transfer function

$$G_{2N}(s) = \left(\frac{K_1}{T_1 s+1} + \frac{K_2}{T_2 s+1}\right)\frac{1}{T_d s+1} \tag{7}$$

$T_1 = 989.7608$; $T_2 = 66.1962$;

$K_1 = 4.7492$; $K_2 = 3.2446$

$T_d = 20$

PI_1-P controller

Without the time delay T_d that characterizes the non-modelled loop dynamics, the output of both channels can be described as

$$\dot{y}_1 = (K_1 u - y_1)/T_1 ; \; \dot{y}_2 = (K_2 u - y_2)/T_2 \tag{8}$$

For $T_d = 0$ and the system output

$$y = y_1 + y_2 \tag{9}$$

one can write

$$\dot{y} = \dot{y}_1 + \dot{y}_2 = \overline{K}u - \frac{1}{T_1} y - \tau y_2 ;$$
$$\overline{K} = \left(\frac{K_1}{T_1} + \frac{K_2}{T_2}\right) ; \; \tau = \left(\frac{1}{T_2} - \frac{1}{T_1}\right) \tag{10}$$

$$e = w - y ; \; \dot{e} = -\dot{y} \tag{11}$$

The control signal u_w that maintains the system output at $w = const$ is

$$u_w = w/(K_1 + K_2) \tag{12}$$

Using this control the steady state outputs of the particular channels are

$$y_{1\infty} = K_1 u_w = wK_1/(K_1 + K_2) ;$$
$$y_{2\infty} = K_2 u_w = wK_2/(K_1 + K_2) \overset{!}{=} w_2 \tag{13}$$

The pole assignment control requires control error dynamics described by

$$\dot{e} = \alpha e \tag{14}$$

whereby α is a chosen closed loop pole. Substituting into (11) one gets

$$-\overline{K}u + (y - w + w)/T_1 + \tau(y_2 - w_2 + w_2) = \alpha(w - y) \tag{15}$$

that corresponds to parallel structure of the P-P controller

$$u = \frac{1}{K} w + K_R e + K_{R2} e_2 ;$$
$$e_2 = w_2 - y_2 ;$$
$$K_R = -\frac{\alpha + 1/T_1}{\overline{K}} ; \tag{16}$$
$$K_{R2} = -\frac{T_1 - T_2}{K_1 T_2 - K_2 T_1}$$

In order to get monotonic setpoint step responses, the closed loop pole is not allowed to take any negative value $\alpha < 0$, but just values from restricted interval

$$\alpha \in (\alpha_e, 0) ; \; \alpha_e T_d = -(1 + T_d/T_1)^2/4 \tag{17}$$

whereby α_e was derived for the dominant 1st order dynamics of the first channel by analyzing conditions of the double real dominant closed loop pole. For the two-channel robust design, the derivation of an appropriate tuning could be much more complex due to the relatively large number of model parameters.

This P-P controller can be expanded to P-PI controller [6,17] by the I-action designed as the disturbance reconstruction and compensation (Fig. 2).

Tuning of the controller is the same as in the PI1 case.

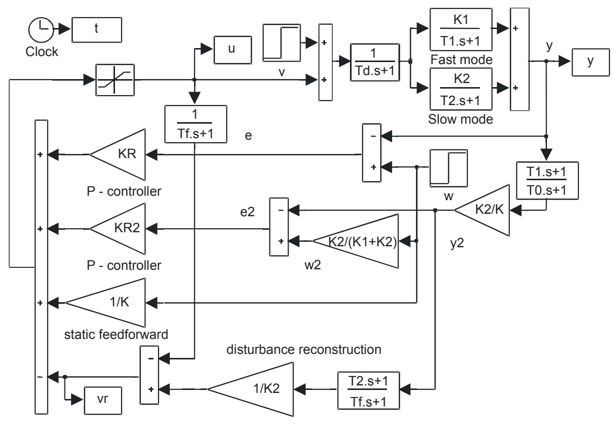

Fig. 2. Simulink model of the control loop with the P-PI controller

5 CONTROL BASED ON 3RD ORDER DOMINANT DYNAMICS

Use of the 3rd order model could be physically motivated by three modes of heat transfer. Since such a model reasonably increases complexity of the resulting controller, experiments should show, how far does its use to improve plant-model matching and how far does it improve the closed loop control.

Again it will be considered that each channel can be described by the first order model and the non-modelled closed loop dynamics is approximated by T_d .

$$G_{3N}(s) = \left(\frac{K_1}{T_1 s + 1} + \frac{K_2}{T_2 s + 1} + \frac{K_3}{T_3 s + 1} \right) \frac{1}{T_d s + 1} \tag{18}$$

$T_1 = 1024.3$; $T_2 = 261.4970$; $T_3 = 54.5900$

$K_1 = 4.5136$; $K_2 = 0.6478$; $K_3 = 2.8383$

$T_d = 20$

PI_I-P-P controller

Let y be the output of the system (18) for $T_d = 0$, then

$$y = y_1 + y_2 + y_3 = \left(\frac{K_1}{T_1 s + 1} + \frac{K_2}{T_2 s + 1} + \frac{K_3}{T_3 s + 1} \right) u = \\ = K \frac{b_2 s^2 + b_1 s + b_0}{(T_1 s + 1)(T_2 s + 1)(T_3 s + 1)} u \tag{19}$$

where u is the control signal and K is

$$K = K_1 + K_2 + K_3 \tag{20}$$

For a piecewise constants setpoint signal, the control error is defined as

$$e = w - y, \dot{e} = -\dot{y} \tag{21}$$

The pole assignment control is defined by the requirement of a regular control error decrease

$$\dot{e} = \alpha e \tag{22}$$

In other words

$$-\dot{y}_1 - \dot{y}_2 - \dot{y}_3 = \alpha (w - y) \tag{23}$$

where α is the chosen closed loop pole. Then one can write

$$-\frac{1}{T_1}(K_1 u - y_1) - \frac{1}{T_2}(K_2 u - y_2) - \frac{1}{T_3}(K_3 u - y_3) = \\ = \alpha (w - y) \tag{24}$$

$$-\overline{K}u + \frac{1}{T_1}(y - y_2 - y_3) + \frac{1}{T_2} y_2 + \frac{1}{T_3} y_3 = \alpha (w - y) \tag{25}$$

whereby

$$\overline{K} = \left(\frac{1}{T_1} K_1 + \frac{1}{T_2} K_2 + \frac{1}{T_3} K_3 \right) \tag{26}$$

Then

$$-\alpha (w - y) + \frac{1}{T_1}(y - w + w) - \frac{1}{T_1}(y_2 + y_3) + \frac{1}{T_2} y_2 + \\ + \frac{1}{T_3} y_3 = \overline{K}u \tag{27}$$

and one can write

$$\overline{K}u = -\left(\alpha + \frac{1}{T_1} \right)(w - y) + \left(\frac{1}{T_1} - \frac{1}{T2} \right) e_2 + \left(\frac{1}{T_1} + \frac{1}{T_3} \right) e_3 + \\ + [K_1 T_2 T_3 + K_2 (T_2 T_3 + T_1 - T_2) + K_3 (T_2 T_3 + T_1 - T_3)] w \tag{28}$$

So the control is

$$u = w[K_1 T_2 T_3 + K_2 (T_2 T_3 + T_1 - T_2) + \\ K_3 (T_2 T_3 + T_1 - T_3)] / \overline{K} + K_R e + K_{R2} e_2 + K_{R3} e_3 \tag{29}$$

which is the parallel P-P-P controller with parameters

$$K_R = -\left(\alpha + \frac{1}{T_1} \right) / \overline{K} \\ K_{R2} = \left(\frac{1}{T_1} - \frac{1}{T_2} \right) / \overline{K} \\ K_{R3} = \left(\frac{1}{T_1} - \frac{1}{T_3} \right) / \overline{K} \tag{30}$$

In practice there will be mostly measured just the main system output. Therefore, the reconstruction of the auxiliary outputs y_2, y_3 is necessary. There are two easy ways to obtain these outputs. They can either be reconstructed from the output of the system, from the control signal, or from both. In this case the reconstruction from the system output will be used again.

The controller can be expanded to PI-P-P by adding the disturbance reconstruction circuitry.

6 EXPERIMENTAL REUSLTS

For comparing the experimental results, the IAE (Integral of Absolute Error) or the ISE (Integral of Squared Error) criteria will be used defined for the control error (21) as

$$IAE = \int_0^\infty |e(t)| dt \tag{31}$$

$$ISE = \int_0^\infty e(t)^2 dt \tag{32}$$

These criteria showed to be useful also when applied to the I-action signal resulting from the DOB. Since it was no outer disturbance acting on the controlled plant, all generated signal may be considered as result of the model imperfection. Let us define these criteria as

$$IAI = \int_0^\infty |i(t) - i_\infty| dt \qquad (33)$$

$$ISI = \int_0^\infty (i(t) - i_\infty)^2 dt \qquad (34)$$

whereby $i(t)$ is the output of the I-action (DOB) and i_∞ is the steady state value of the I-action.

Besides of these criteria applied to the control error to characterize plant-model matching at their outputs, another criterion [25] called Total Variance (TV) was used for characterizing "smoothness" at the controller output. This was defined as

$$TV = \int_0^\infty \left|\frac{du}{dt}\right| dt \approx \sum_i |u_{i+1} - u_i| \qquad (35)$$

Since it is difficult to be experimentally evaluated continuously, it is usually computed after appropriate discretization with sampling period as small as possible.

PI_1-controller

Here, the real-time experiment is compared with two simulations using PI_1-controller. In the first one, the plant is represented by model (1), in the second one by model (18). As the simulation shows, the 3rd order model is in relatively good concurrence with real plants dynamics. The parameters of the controller are $\alpha = \alpha_e/1.3 = \text{-}0.009, T_f = -1/\alpha = 111$.

While the control sequence corresponding to the 1st order model is close to the required one pulse at the saturation + exponential transition to the steady state. Due to the zero dynamics of real plant, control sequence corresponding to the 3rd order model and real-time control has more complex shape. Also the I-actions have similar transients. The experiment results are in Fig. 3 and Fig. 4

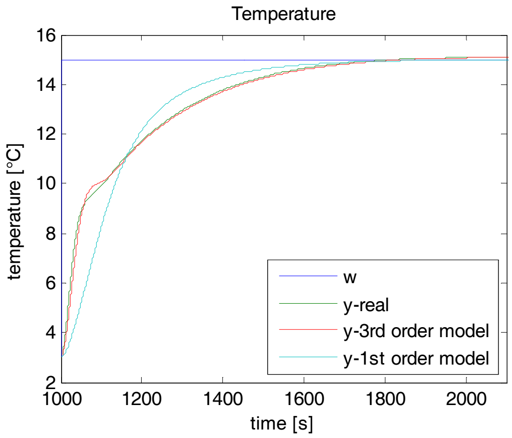

Fig. 3 Temperature

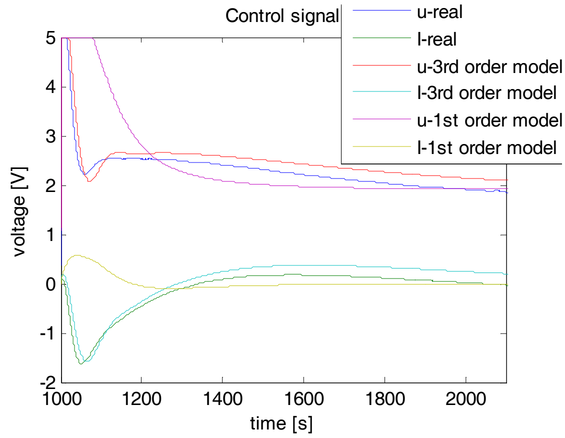

Fig. 4 Control signal

PI_1-P-controller

In this experiment PI_1-P-controller was used for controlling models (7) and (18) and for controlling real experiment. As the simulation shows (Fig. 5,6), the 2nd and the 3rd order models describe the plants dynamics almost in the same way. The equivalent pole is used to compute the closed loop pole as it was in the previous controller. The dominant time constant is used in equation (5).

IAI and ISI indexes for the I-actions are smaller than in the previous experiment.

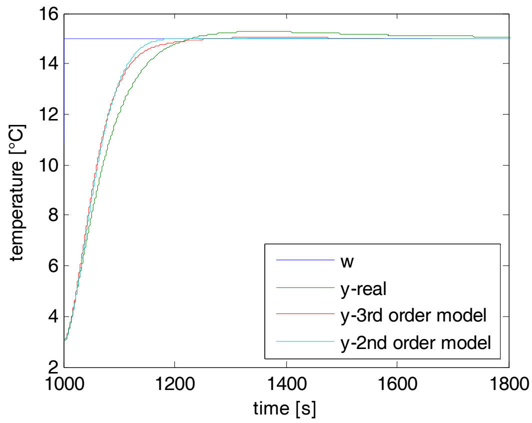

Fig. 5 Temperature

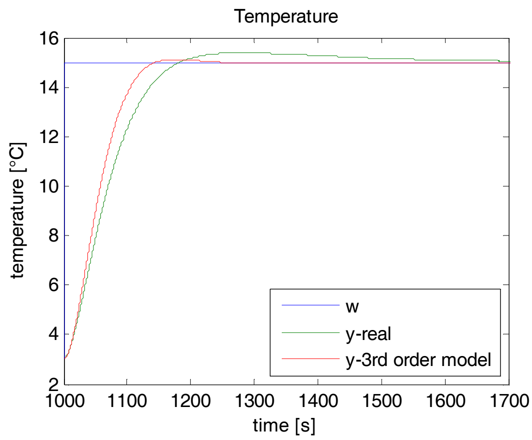

Fig. 7 Temperature

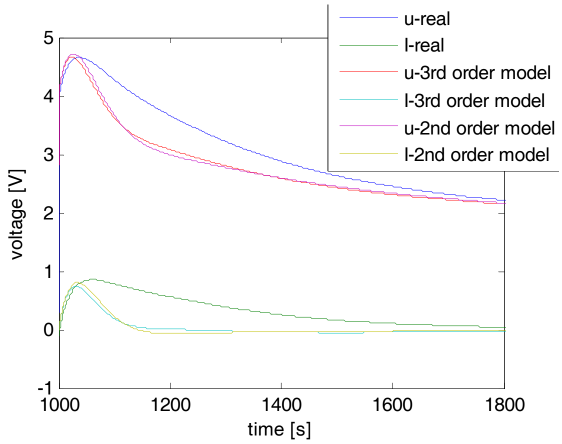

Fig. 6 Control signal

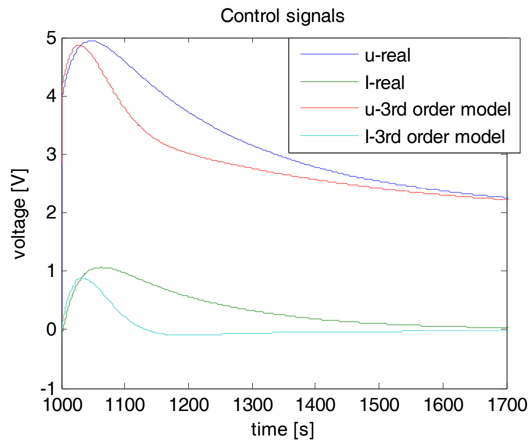

Fig. 8 Control signal

PI_1-P-P-controller

In this experiment the PI_1-P-P-controller was applied to control real experiment and simulation (Fig 7,8). The equivalent pole is used to compute the closed loop pole. The dominant time constant is used in equation (5).

IAE and ISE indexes values are, however, reasonably smaller than values achieved in real experiments with controllers based on models (1) and (7).

Results of the real-time experiments focusing on the plant output behavior that determines also the control error values are summarized in Tab. I. The closed loop behavior reasonably improves by considering two channels of heat transfer instead of one. Further increase of the model complexity leads just to less intensive quality increase.

Statistical comparison

Table I Statistical comparison

	PI_1	**PI_1-P**	**PI_1-P-P**
ISE	8567,7	6426,4	6374,9
IAE	1817,7	986,52	946,79
ISI	235,42	150,7	190,16
IAI	499,76	316,09	303,61
TV	9,4143	7,4222	7,8974

7 CONCLUSION

From the results of the simulations and the real experiments there is hard to decide which model to use. The comparison made in Table I. shows that the 2^{nd} and the 3^{rd} order models resulted in controllers that gave rather similar results. So, if one takes into account also the simplicity of the model and of the controller, the model (7) and the PI_1-P-controller would be the most suitable choice. The higher order models would be efficient only if already negligible improvement in performance would bring reasonable profit increase.

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