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ROBUST DECENTRALIZED CONTROLLER DESIGN WITH SPECIFIED PHASE MARGIN

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Abstract: This paper presents the robust decentralized controller design in the frequency domain for stable plants. Robust condition based on M-delta structure is included in controller design. In controller design for MIMO systems equivalent subsystem method is used. For subsystems of equivalent model, frequency method ensuring desired phase margin is applied. Design procedure is illustrated on two tanks process.

1. INTRODUCTION

PID controllers are standard and well-proven solution for the majority of industrial applications. Over the years, a plenty of PID tuning rules were developed see e.g. (Šulc and Vítečková, 2005). In this paper decentralized PID controller design approaches are developed for stableand unstable systems and extended to satisfy robust stability conditions in terms of unstructured uncertainty developed in (Kozáková A., Veselý, 2005. Controllers for subsystems are designed for specified phase margin.

The paper is organized as follows: preliminaries and problem formulation are given in Section 2, robust stability conditions, ESM, and robust design procedure in Section 3. In Section 4 is a detailed robust decentralized PID controller design procedure illustrated on two tanks process. Conclusions are drawn at the end of the paper.

2. PRELIMINARIES AND PROBLEM FORMULATION

In frequency domain robust controller design very often consists of two steps: controller design and robust condition verification. If robust condition is not passed than controller is redesigned and condition is verified again, so these types of approaches are iterative.

Aim of this paper is to develop robust controller design method which will consist of only one step. So the robust stability condition will be included into the design procedure so that the designed controller will ensure robust stability without any iteration.

3. THEORETICAL RESULTS

3.1 Robust stability conditions

When designing a controller a major source of difficulty is plant model inaccuracy; hence uncertainty models are to be used which means that instead of a single model a class Π of perturbed models is to be considered. Denote $\widetilde{G}(s) \in \Pi$ any perturbed plant model and $G(s) \in \Pi$ the nominal plant model. A simple uncertainty model is obtained using unstructured uncertainty $\Delta(s)$. Commonly used uncertainty forms are: additive (a) and input multiplicative (i) uncertainties

Standard feedback configuration with unstructured uncertainty of any type can be rearranged to obtain the general $M-\Delta$ structure in Fig. 1 where $M(s)$ represents the nominal model and $\Delta(s) : \sigma_{\max}[\Delta(j\omega)] \le 1$ the normalized perturbation.

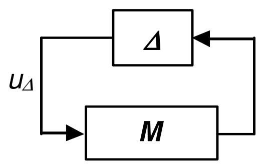

Fig. 1 – $M-\Delta$ structure

Robust stability condition for unstructured perturbations is formulated in terms of stability of the $M-\Delta$ system: if both the nominal system $M(s)$ is stable (nominal stability) and the normalized perturbation $\Delta(s)$ is stable, closed-loop stability is guaranteed for

$$\sigma_{\max}[M(j\omega)] < 1\ , \quad \forall\omega \qquad (1)$$

For individual uncertainty types $M = l_k M_k$, $k = a, i$ in particular

- for additive uncertainty

$$M_a(s) = -[I + R(s)G(s)]^{-1} R(s)$$
$$l_a(\omega) = \max_{\tilde{G}\in\Pi} \sigma_{\max}[\tilde{G}(s) - G(s)] \qquad (2)$$

- for input multiplicative uncertainty

$$M_i(s) = -[I + R(s)G(s)]^{-1} R(s)G(s)$$
$$l_i(\omega) = \max_{\tilde{G}\in\Pi} \sigma_{\max}\{G^{-1}(s)[\tilde{G}(s) - G(s)]\} \qquad (3)$$

In view of (2), (3), condition (1) reads as follows

$$\sigma_{\max}(M_k(s)) < \frac{1}{|l_k(s)|}\ ,\ k = a, i \qquad (4)$$

In the robust stability condition for input multiplicative uncertainty $M_i(s)$ represent the complementary sensitivity function.

$$M_i(s) = -[I + R(s)G(s)]^{-1} R(s)G(s) = -T(s) \quad (5)$$

Denote right side of inequality (4) as $U(s)$.

$$U(s) = \frac{1}{|l_i(s)|} \qquad (6)$$

$U(s)$ does not depend on controller $R(s)$ so it can be calculated before controller design.

Similar it is for additive uncertainty where inequality (4) is rearrange into following form:

$$\sigma_{\max}(M_a(s)G(s)) < \frac{G(s)}{|l_a(s)|} \qquad (7)$$

Than

$$M_a(s)G(s) = T(s) \qquad (8)$$

And

$$U(s) = \frac{G(s)}{|l_a(s)|} \qquad (9)$$

If controller $R(s)$ is designed so that maximum value M_t of complementary sensitivity function $T(s)$ is smaller than minimal value of $U(s)$, system with designed controller satisfy robust stability condition (1).

This can be reached using any frequency controller design method ensuring desired phase margin (PM), for SISO systems because (Skogestad and Postletwaithe, 1997):

$$PM \geq 2\arcsin\left(\frac{1}{2M_t}\right) \qquad (10)$$

For MIMO systems due to interactions is ensuring of desired phase margin in subsystems more complicated.

Hence in this paper for MIMO systems equivalent subsystem method (Kozaková, et al., 2009) will be used for transforming nominal model $G(s)$ into diagonal model of equivalent subsystems $G^{eq}(s)$. For subsystems of equivalent model $G^{eq}(s)$, SISO method ensuring desired phase margin will be used.

3.2 Equivalent subsystem method

Equivalent subsystem method will be used to simplify the full nominal model matrix into diagonal equivalent one. Subsystems in equivalent matrix are called equivalent subsystems and are calculated with taking account into interactions.

In this paper only equations necessary for equivalent subsystem calculation will be written. More details about this method are in (Kozaková, et al., 2009).

Full matrix of nominal model $G(s)$ can be split into matrix containing diagonal elements $G_d(s)$ of $G(s)$ and $G_m(s)$ containing off-diagonal elements.

$$G^{eq}(s) = G_d(s) - P(s) = diag\{G_i(s) - p_i(s)\}_{i=1,2,...,m} =$$
$$= diag\{G_i^{eq}(s)\}_{i=1,2,...,m} \qquad (11)$$

G_i^{eq} is a diagonal matrix of equivalent subsystems.
For individual subsystems, (11) yields

$$1 + R_i(s)G_i^{eq}(s) = 0 \qquad i = 1,2,...,m \qquad (12)$$

which are the m equivalent characteristic equations.

In the context of the independent design philosophy, the design parameters $p_i(s),\ i = 1,2,\ldots,m$ represent the bounds for individual designs. To be able to provide closed-loop stability of the full system using a DC controller, $p_i(s),\ i = 1,2,\ldots,m$ are to be chosen so as to cope with the interactions $G_m(s)$.

A general method for choosing $P(s)$ is not available yet, however interesting results have been obtained for the case when

$$P(s) = p_i(s)I \qquad (13)$$

with identical entries. So $p_i(s)$ will be choose equal to one of the m characteristic functions $g_i, i = 1,2,...m$ of $[-G_m(s)]$.

$$p_i(s) = -g_k,\ k \in \{1,2,...,m\} \qquad (14)$$

3.3 *Robust controller design procedure*

1. Calculation of nominal model $G(s)$;
2. Calculation of $U(s)$ according (6, 9);
3. Set of M_t as minimal value of $U(s)$;
4. Minimal phase margin calculation according (10);
5. Equivalent model $G^{eq}(s)$ calculation according (14, 13, 11);
6. Controllers design for equivalent model subsystems using SISO method ensuring phase margin greater than calculated in point 4.;

Note: If controlled system is SISO point no.5 is omitted and controller is designed for nominal model $G(s)$.

4. CASE STUDY

Consider two tanks process with two inputs (pumps voltage) and two outputs (water level) depicted in figure 2.

Fig.2 Two tanks process

Process contains also valves which are used for decreasing water level in tanks. Valves voltage can be changed in range (0-10V). Different operating points were obtained using identification with valves at 7.5V, 8.5V and 9.5V. This system has normally no interactions so software interactions were added according to fig. 3.

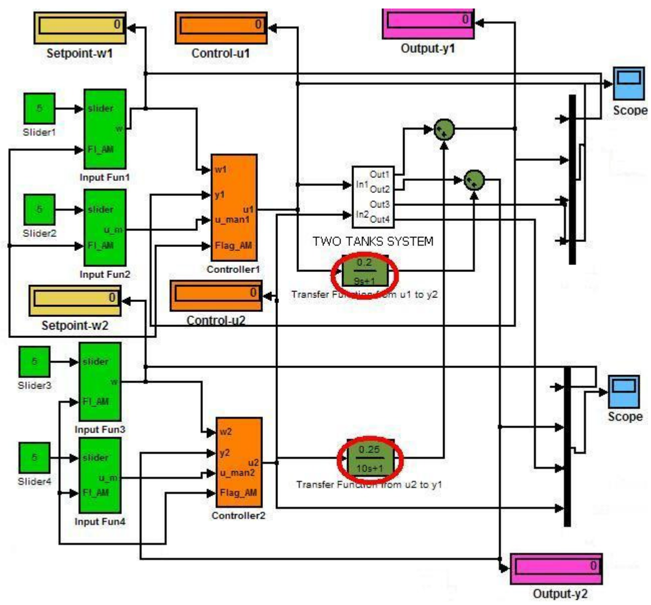

Fig. 3 Simulation diagram with interactions

$$\tilde{G}_1(s) = \begin{bmatrix} \dfrac{0.049s+0.98}{109.2s^2+11.83s+1} & \dfrac{0.25}{10s+1} \\ \dfrac{0.2}{9s+1} & \dfrac{0.059s+1.19}{27.78s^2+10.91s+1} \end{bmatrix}$$

$$\tilde{G}_2(s) = \begin{bmatrix} \dfrac{0.049s+0.97}{46.5s^2+8.76s+1} & \dfrac{0.25}{10s+1} \\ \dfrac{0.2}{9s+1} & \dfrac{0.048s+0.96}{61.06s^2+11.41s+1} \end{bmatrix} \quad (15)$$

$$\tilde{G}_3(s) = \begin{bmatrix} \dfrac{0.047s+0.94}{38.46s^2+8.21s+1} & \dfrac{0.25}{10s+1} \\ \dfrac{0.2}{9s+1} & \dfrac{0.044s+0.88}{31.2s^2+5.15s+1} \end{bmatrix}$$

Nominal model of this plant was calculated as average value of $\tilde{G}_1(s)$, $\tilde{G}_2(s)$, $\tilde{G}_3(s)$.

$$G(s) = \begin{bmatrix} \dfrac{0.048s+0.96}{64.71s^2+9.6s+1} & \dfrac{0.2}{9s+1} \\ \dfrac{0.25}{10s+1} & \dfrac{0.05s+1}{40.01s^2+9.16s+1} \end{bmatrix} \quad (16)$$

In this example additive uncertainty will be used so $U(s)$ calculates according (9) and it is depicted in fig. 4.

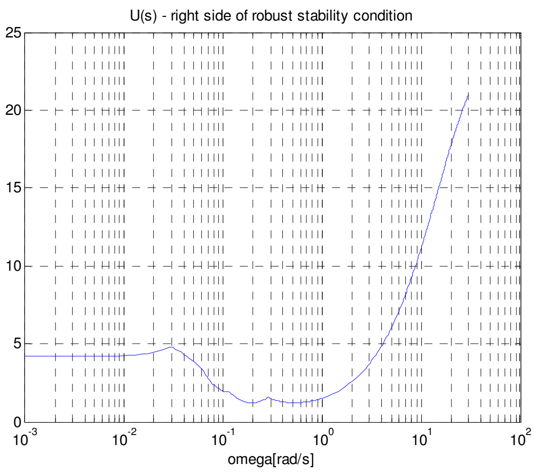

Fig.4 Behavior of $U(s)$

Minimal value of $U(s)$, $U_{\min}(s) = 1.205$ is set as M_t which can be recalculate according (10) into minimal phase margin $PM_{\min} = 49$ which presumably ensures robust stability.

From nominal model equivalent model will be calculated. At first characteristic functions are calculated from $G_m(s)$ (Fig. 5).

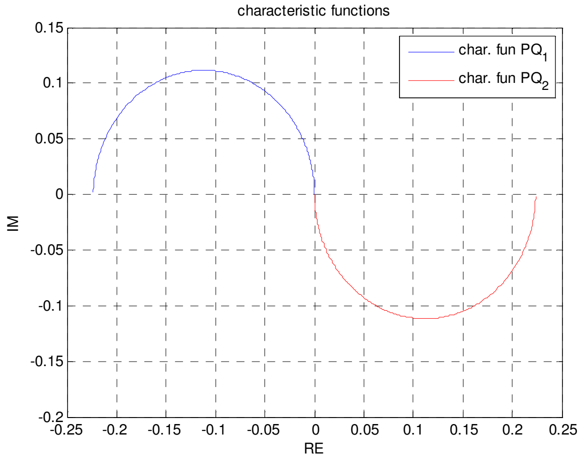

Fig.5 Characteristic functions PQ_1 and PQ_2

For equivalent subsystems calculation according (14, 13 and 11) we use PQ_2. Equivalent subsystems are depicted in fig. 6,7.

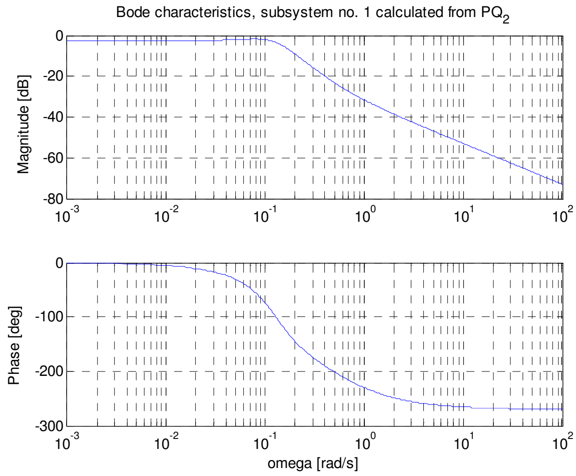

Fig. 6 Bode plot of equivalent subsystem no.1

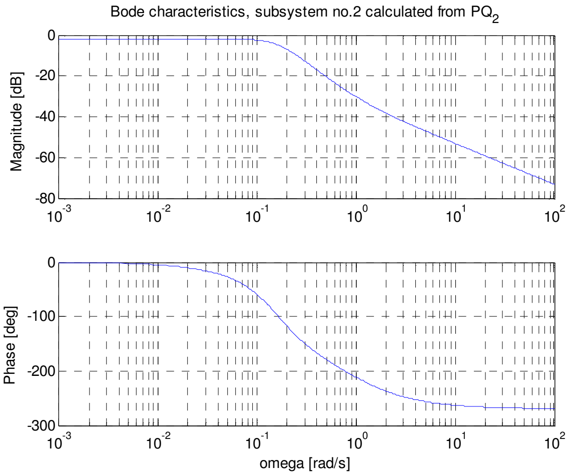

Fig. 7 Bode plot of equivalent subsystem no.2

For each subsystem controller will be calculated. Aim of the controllers design is to fulfill robust conditions and for nominal model have overshoot less than 15%.

Phase margin corresponding to overshoot 15% (Fig. 8) is approximately $PM = 60° > PM_{\min}$.

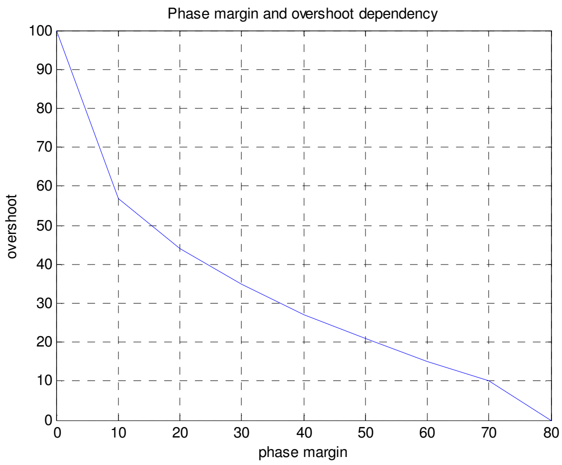

Fig. 8 Dependency of phase margin and overshoot

For subsystem 1 was designed controller with following parameters,

$$r_{11}(s) = 0.558 + \frac{0.09}{s} + 0.284s \qquad (17)$$

and for subsystem 2 controller with parameters:

$$r_{22}(s) = 1.03 + \frac{0.13}{s} + 0.41s \qquad (18)$$

Decentralized controller $R(s)$ consists of controllers for both subsystems.

$$R(s) = \begin{bmatrix} r_{11} & 0 \\ 0 & r_{22} \end{bmatrix} \qquad (19)$$

Reaching of desired crossover frequency and phase margin for both subsystems proofs Fig. 9,10.

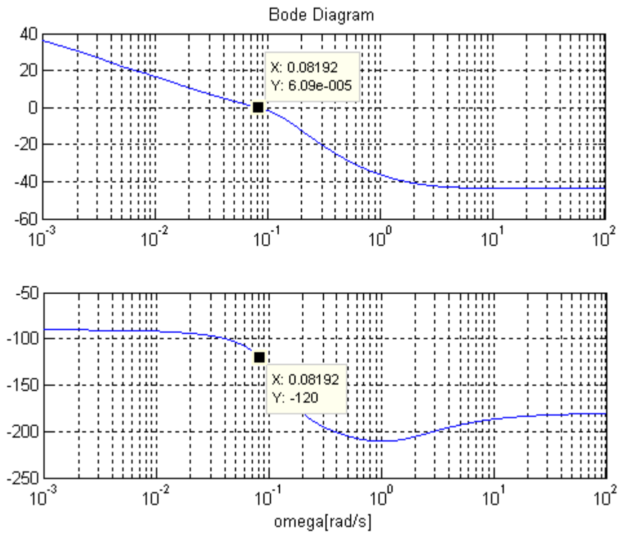

Fig. 9 Bode characteristics for subsystem 1 with PID

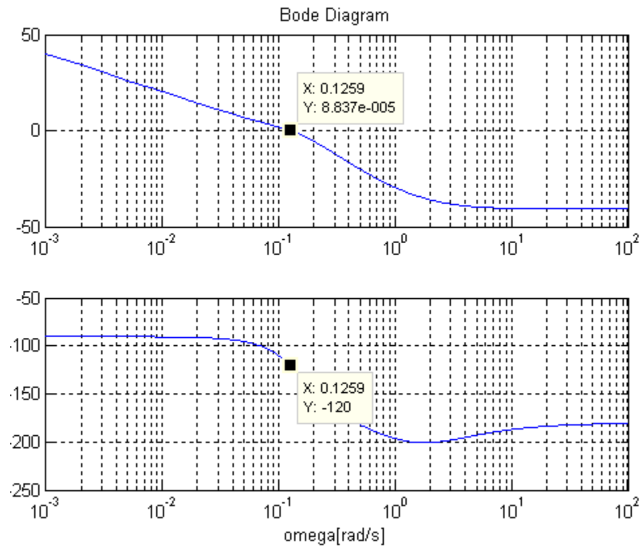

Fig. 10 Bode characteristics for subsystem 2 with PID

Matlab simulation in nominal model (Fig. 11) show that system has overshoot less than 15% by step change in both outputs. So we can see that using controllers with desired phase margin for equivalent subsystems it is possible to reach desired overshoot for nominal model.

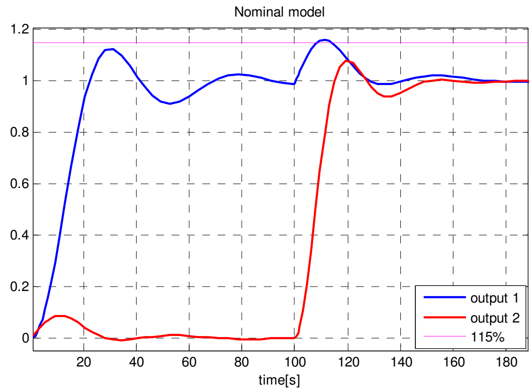

Fig. 11 Nominal model simulation

Because our process is stable, negative roots in operating points (20) proofs stability in this points and robust stability condition (Fig. 12) show that process with designed controller is also robust stable.

$$\Lambda_1 = \{-0{,}14 \pm 0.15i; -0.2 \pm 0.1i; -0.12; -0.07\}$$
$$\Lambda_2 = \{-0.06 \pm 0.15i; -0.05 \pm 0.14i; -0.1 \pm 0.015i; -0.064\}$$
$$\Lambda_3 = \{-0.055 \pm 0.22i; -0.07 \pm 0.15i; -0.09 \pm 0.015i; -0.053\} \quad (20)$$

Finally designed controllers were set on the real process. Step responses in all operating points are depicted in fig. 13.

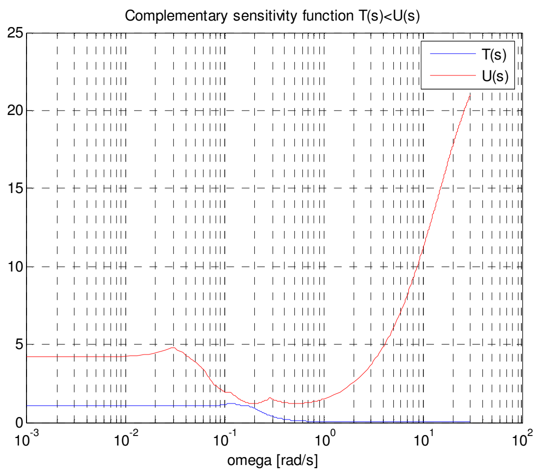

Fig. 12 Robust stability condition

Real process experiments shows that overshoot is really less than 15%, but the step response is different from Matlab simulation. It is due to inaccuracy of models obtained by identification.

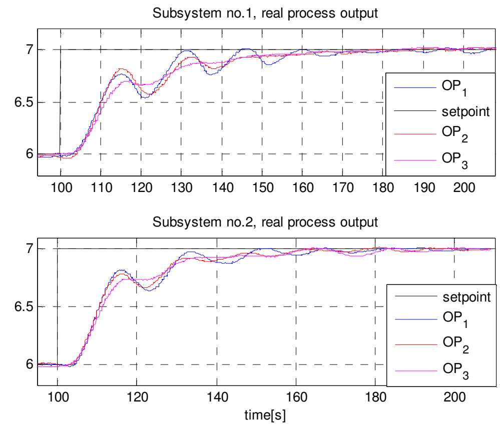

Fig. 13 Step responses of real process

5. CONCLUSION

The paper deals with the robust decentralized controller design in the frequency domain for stable and unstable plants. Equivalent subsystem method was used to simplify the full nominal model matrix into diagonal equivalent one. Controllers were designed for subsystems of equivalent matrix independently, so that desired phase margin in equivalent subsystems guaranteed overshoot for outputs in nominal model. Controller design process was illustrated on two tanks example.

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REFERENCES

Kozáková A., Veselý V. (2005) *A frequency domain design technique for robust decentralized controllers*. Preprints of the 16th IFAC World Congress. Prague, Czech Republic July 3-8, p. Mo-E21-TO/6.

Kozaková, A., Veselý, V., Osuský, J. (2009). A new Nyquist- based technique for tuning robust decentralized controllers, *Kybernetika*, vol.45, No 1., pp. 63-83

Skogestad, S., Postlethwaite, I. (1997): Multivariable Feedback Control: Analysis and Design. *John Wiley and Sons*

Šulc B., Vítečková, M., (2004) *Theory and practice of control loops design*. ČVUT, Praha (in Czech)