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ROBUST PID CONTROLLER DESIGN FOR COUPLED-TANK PROCESS

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Abstract: The paper deals with the design of the robust PID controller for real uncertain Coupled-Tank process in the frequency domain. Only the first independent tank is considered (single-input single-output system). Robust controller is designed in two ways. The first approach is performed with the Edge Theorem and the Neimark's D-partition method for the affine model and the second one is performed with the modification of the Neimark's D-partition which ensures desired phase margin.

Keywords: SISO, Robust PID Controller, Edge Theorem, D-partition, phase margin.

1. INTRODUCTION

Control of real processes inherently includes uncertainties (modeling errors due to linearization and approximation, disturbances etc.), which have to be considered in the adequate control design. Therefore robustness belongs to an important control design qualities: closed loop system stability and performance should be guaranteed over the whole uncertainty domain, (Vesely et al. 2006).

There exist various approaches to robust stability analysis and robust control design for uncertain linear systems. In this paper the frequency domain PID controller design for real Coupled-Tank process is considered. Liquid tank processes play important role in industrial application such as in food processing, filtration, pharmaceutical industry, water publication system, industrial chemical processing and spray coating (Ramli et al. 2009). Many industrial applications are concerned with level of liquid control, may it be a single loop level control or sometimes multi loop level control (Ramzad et al. 2008). In this paper only the first tank with liquid is used (SISO).

The paper is organized as follows. The next section gives details about Coupled-Tank process. Section 3 introduces a PID controller design using two approaches. In section 4 some results of robust PID controller design are presented. Several step responses of closed-loop system with proposed PID controller are plotted there. Finally, conclusion is given in section 5.

2. COUPLED-TANK PROCESS

The industrial Coupled-Tank process is one of the real processes built for control education and research at Institute of Control and Industrial Informatics. The apparatus consists of two tanks (T1 and T2 in Fig. 1), which can be coupled using valve V12 (the manual valve). Therefore the Coupled-Tank process with two tanks represents a multi-input multi-output (MIMO) system for opened valve V12 or two independent single-input single-output (SISO) systems for closed valve V12. Both tanks are made of Plexiglas. These two tanks are mounted on a platform with a metering scale before each tank indicating the approximate liquid level in tank. Exact liquid level in each tank is measured using an electronic sensor. Other components of system are liquid basin (reservoir), two pumps (Pump1 and Pump2 in Fig. 1), two outlet valves (V1 and V2 in Fig. 1) and electronic circuit communicating with *LABREG* software in computer. This software is made for identification and control of real processes. The *LABREG* operates in MATLAB using toolboxes *SIMULINK*, *Ident*, *Control* and *Real Time*. Cooperation between Coupled-Tank process and computer and *LABREG* software is ensured using Advantech data acquisition card of type PCI 1711. More about *LABREG* and mentioned toolboxes can be found in (Kajan et al. 2007).

The paper deals with design of robust controller for SISO system (valve V12 is closed), consequently there can be used one or two independent tanks. Only one tank process is considered, therefore the purpose is to control liquid level in the first tank by the inlet liquid flow from the first electronic DC pump (Pump1). The process input is $u_1(t)$ (voltage input to Pump1) and the output is $h_1(t)$ (liquid level in the first tank – T1). Input power is bounded by interval $\langle 0,10\rangle$ volts and output signal is measured using electronic sensor. Q_{i1} and Q_{o1} in Fig. 1 denote the inlet and outlet flow rates for T1 respectively. Outlet flow is affected by electronic outlet valve (V1), which can be set manually from 0 to 10 volts (for 0 [V] is closed, for 10 [V] fully opened) and represent perturbation.

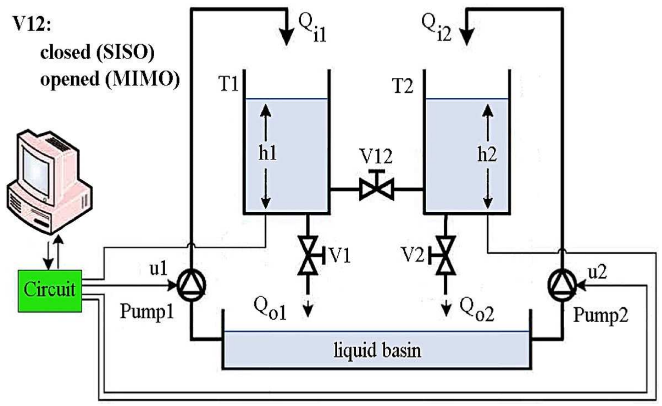

Fig. 1. Coupled-Tank process

3. PRELIMINARIES AND PROBLEM FORMULATION

3.1 Robust controller design using the Edge Theorem

For this theory affine model of the plant is used. It is used advantageously because a part of parameters of the real process vary dependently. Affine model is in this form

$$G(s)=\frac{b_0(s)+\sum_{i=1}^{p}q_ib_i(s)}{a_0(s)+\sum_{i=1}^{p}q_ia_i(s)} \tag{1}$$

where $b_0(s),\ b_i(s)$ and $a_0(s),\ a_i(s)$ are polynomials of numerator and denumerator and uncertain parameters q_i are from interval $\left[\underline{q_i},\overline{q_i}\right]$.

Each real uncertain parameter q_i varies within a p-dimensional domain. In other words, the parameter vector $q^T=\left[q_1,\dots,q_p\right]$ varies in the hypercube (Ackerman 1997, Bhattacharyya et al. 1995)

$$Q=\left\{q\mid q_i\in\left[\underline{q_i},\overline{q_i}\right],\ i=1,2,...,p\right\} \tag{2}$$

Alternating minimal ($\underline{q_i}$) and maximal ($\overline{q_i}$) value of q_i , we obtain the polytope with 2^p vertices. Each vertex can be represented by a transfer function with constant coefficients. Transfer function (1) describes a polytopic system.

Consider the controller described by transfer function

$$G_R(s)=\frac{F_1(s)}{F_2(s)} \tag{3}$$

where $F_1(s)$ and $F_2(s)$ are polynomials with constant parameters.

If parameter q varies within a hypercube, it generates a polytopic family of closed-loop characteristic polynomials described as follows

$$p(s,q)=b_0(s)F_1(s)+a_0(s)F_2(s)+\sum_{i=1}^{p}q_i\left[b_i(s)F_1(s)+a_i(s)F_2(s)\right] \tag{4}$$

or in more general form according to (Hypiusová et al. 2007, Hypiusová et al. 2008)

$$p(s,q)=p_0(s)+\sum_{i=1}^{p}q_ip_i(s) \tag{5}$$

where $q_i\in Q$.

Theorem 1 - Edge Theorem (Hypiusová et al. 2007)

The polytopic family of characteristic polynomials (5) is stable if and only if the edges of set Q are stable.

The Edge Theorem gives an elegant solution to the problem of determining the root space of polytopic systems. Therefore the robust stability of such systems can also be determined (Bhattacharyya et al. 1995). The stability condition for polytopic family of characteristic polynomials (5) is given in the following theorem using robust Hurwitz stability criteria. Using the Bialas Theorem stability of each edge of the polytopic box can be checked.

Theorem 2 - Bialas Theorem (Hypiusová et al. 2007)

The polynomial family

$$p(s,Q)=\left\{\lambda p_a(s)+(1-\lambda)p_b(s),\ \lambda\in[0,1]\right\} \tag{6}$$

is stable if and only if:

- $p_a(s),\ p_b(s)$ are stable,
- the matrix $\left(H_n^{(b)}\right)^{-1}H_n^{(a)}$ has no nonpositive real eigenvalues

where matrices $H_n^{(b)}$ and $H_n^{(a)}$ are Hurwitz matrices of following polynomials

$$\begin{aligned}p_b(s)&=p_{b0}+p_{b1}s+\ldots+p_{bn}s^n\ ,\ \ p_{bn}>0\\ p_a(s)&=p_{a0}+p_{a1}s+\ldots+p_{an}s^n\ ,\ \ p_{an}>0\end{aligned} \tag{7}$$

By applying the Neimark's D-partition method with Edge Theorem, the required stability degree of closed-loop system can be guaranteed. The controller coefficients are chosen so that the vertices and edges of polytopic system are stable.

3.2 Robust controller design with desired phase margin

This approach is in details described in (Hypiusová et al. 2010a, Hypiusová et al. 2010b), where closed-loop system with $G_R(s)$ (transfer function of PID controller) and $G(s)$ (transfer function of the real plant) is considered. The real perturbed plant with unstructured inverse additive uncertainties is described as follows (Vesely et al. 2006)

$$G(s)=G_0(s)(I+w_{ia}(s)\Delta_{ia}(s)G_0(s))^{-1} \tag{8}$$

where $G_0(s)$ is nominal model, $w_{ia}(s)$ is stable weighting scalar transfer function, $\Delta_{ia}(s)$ is normalized matrix of unstructured uncertainty ($\Delta_{ia}(s) \le 1$).

Weighting scalar transfer function must be chosen for all ω in accordance with

$$\left|w_{ia}(\omega)\right| \ge l_{ia}(\omega) \tag{9}$$

and weighting function $l_{ia}(\omega)$ is the maximum singular value of difference $G_0(j\omega) - G_k(j\omega)$ for N ($k = 1,\dots,N$) known transfer functions:

$$l_{ia}(\omega) = \max_k \sigma_M (G_0(jw) - G_k(jw)) \tag{10}$$

The nominal model stability is equivalent to the stability of the $M\Delta$-structure. We thus need to derive the robust stability conditions using $M\Delta$-structure for checking the stability according to (Hypiusová et al. 2010a, Skogestad et al. 2005) as follows

$$\sigma_M(M_0(s)) < \frac{1}{l_{ia}(\omega)} \tag{11}$$

and

$$M_0(s) = \frac{G_0(s)}{1 + G_R(s)G_0(s)} \tag{12}$$

where $G_R(s)$ and $G_0(s)$ are transfer functions of PID controller and nominal model. Nominal model has in this case the following form

$$G_0(s) = \frac{B_0(s)}{A_0(s)} = \frac{\left(\sum_{i=1}^{N} B_i(s)\right)\Big/N}{\left(\sum_{i=1}^{N} A_i(s)\right)\Big/N} \tag{13}$$

where B_i, A_i $(i = 1,\dots,N)$ are polynomials of numerator and denumerator of N identified transfer functions of the real process (in N working points).

Consider transfer function of PID controller

$$G_R(s) = \frac{K_D s^2 + K_P s + K_I}{s} = K_P + \frac{K_I}{s} + K_D s \tag{14}$$

The robust PID controller design is performed with the modification of the Neimark's D-partition which ensures stability and desired phase margin of the closed-loop system with nominal model described in (13) as in (Hypiusová et al. 2010a).

The closed-loop system characteristic equation for nominal model is

$$1 + G_R(s)G_0(s) = 0 \tag{15}$$

From (15) the relationship between $G_R(s)$ and $G_0(s)$ can be obtained

$$G_R(s) = -\frac{1}{G_0(s)} \;\Rightarrow\; K_P + \frac{K_I}{s} + K_D s = -\frac{A_0(s)}{B_0(s)} \tag{16}$$

Using substitution $s = j\omega$, real and imaginary part of equation (16) are

$$\begin{aligned} \mathrm{Re}: K_P &= -\frac{A_0(j\omega)}{B_0(j\omega)} \\ \mathrm{Im}: -\frac{K_I}{\omega} j + K_D j\omega &= -\frac{A_0(j\omega)}{B_0(j\omega)} \end{aligned} \tag{17}$$

The D-curve in the complex plane $\mathbb{C}$ for parameter K_P can be plotted from real part (17) by changing value of ω step by step in interval $(0,\infty)$. Similarly it is with imaginary part (17), from where D-curve for parameters K_I and K_D can be plotted. Parameters of PID controller are obtained in two steps. In the first one it is possible to plot D-curve for K_{P1} and K_D (PD controller is obtained) and in the second one for parameters K_{P2} and K_I (PI controller).

When a phase margin is considered, the closed-loop system characteristic equation (15) can be rewritten according to (Hypiusová et al. 2010a, Hypiusová et al. 2010b)

$$1 + G_R(s)G_0(s)e^{-j\varphi} = 0 \tag{18}$$

where φ is the angle of desired rotation in radians (phase margin) and in this way it is possible to rotate the frequency plot. From (18) real and imaginary parts can be obtained, which describe the D-curves as

$$\begin{aligned} \mathrm{Re}: K_P &= -\frac{A_0(j\omega)}{B_0(j\omega)e^{-j\varphi}} \\ \mathrm{Im}: -\frac{K_I}{\omega} j + K_D j\omega &= -\frac{A_0(j\omega)}{B_0(j\omega)e^{-j\varphi}} \end{aligned} \tag{19}$$

Parameters of PD and PI controller are chosen from plotted D-curves. The final PID controller is represented as series connection of PD and PI controller and can be calculated as follows

$$\begin{aligned} G_R(s) &= \left(K_{P1} + K_D s\right)\left(K_{P2} + \frac{K_I}{s}\right) \\ &= \frac{K_{P2}K_D s^2 + \left(K_{P1}K_{P2} + K_D K_I\right)s + K_{P1}K_I}{s} \end{aligned} \tag{20}$$

The first controller (PD) is used for stabilization of system and the second one (PI) ensures desired phase margin.

4. DESIGN OF ROBUST PID CONTROLLER FOR COUPLED-TANK PROCESS

In this case, system step response is examined. Transfer function of the system in all three working points is obtained from the output step response of open loop system using BJ (Box-Jenkins) method of identification. More about BJ method of identification can be found in (Pintelon et al. 2006a, Pintelon et al. 2006b). We consider transfer functions of a liquid level in the first tank (see Fig. 1) obtained by identification in three working points:

WP1 (working point 1):

$$water\ pump\ voltage\ (input\ voltage) = 2,5\ [V]$$
$$step\ of\ water\ pump\ voltage\ in\ time\ t\ to\ 2,75\ [V]$$
$$outlet\ valve\ voltage\ (perturbation) = 7\ [V]$$

Transfer function is obtained by BJ method of identification as

$$G_{WP1}(s) = \frac{2,9676s + 29,9239}{816,9049s^2 + 202,6853s + 1} \quad (21)$$

WP2 (working point 2):

$$water\ pump\ voltage\ (input\ voltage) = 3,5\ [V]$$
$$step\ of\ water\ pump\ voltage\ in\ time\ t\ to\ 4,5\ [V]$$
$$outlet\ valve\ voltage\ (perturbation) = 9\ [V]$$

Transfer function is obtained by BJ method of identification as

$$G_{WP2}(s) = \frac{0,7824s + 7,8848}{360,5413s^2 + 82,2612s + 1} \quad (22)$$

WP3 (working point 3):

$$water\ pump\ voltage\ (input\ voltage) = 4\ [V]$$
$$step\ of\ water\ pump\ voltage\ in\ time\ t\ to\ 5\ [V]$$
$$outlet\ valve\ voltage\ (perturbation) = 10\ [V]$$

Transfer function is obtained by BJ method of identification as

$$G_{WP3}(s) = \frac{0,5461s + 5,5007}{297,473s^2 + 63,8584s + 1} \quad (23)$$

Transfer function of the nominal model is obtained by (13) from the above three working points

$$G_0(s) = \frac{1,431s + 14,44}{491,6s^2 + 116,3s + 1} \quad (24)$$

The respective polytopic (affine) model of the Coupled-Tank process is described by

$$G(s) = \frac{b_0(s) + q_1 b_1(s) + q_2 b_2(s)}{a_0(s) + q_1 a_1(s) + q_2 a_2(s)} \quad (25)$$

where $q_i,\ i = 1,\ldots,N$ are uncertain coefficients and polynomials of numerator and denumerator are

$$b_0(s) = 1,75685s + 17,7123$$
$$b_1(s) = -0,11815s - 1,1921$$
$$b_2(s) = -1,0926s - 11,0195$$
$$a_0(s) = 557,1889s^2 + 133,2719s + 1$$
$$a_1(s) = -31,5341s^2 - 9,2014s$$
$$a_2(s) = -228,1818s^2 - 60,21205s$$

More about practical procedure on how to get the values of polynomials of numerator and denumerator can be found in (Vesely et al. 2006).

4.1 Robust controller design using the Edge Theorem

The robust PID controller is proposed using the Edge Theorem approach for the polytopic model defined in (25). The required degree of stability α is 0. Using Neimark's D-partition method the robust PID controller is designed

$$G_R(s) = \frac{1,6s^2 + 4s + 0,1}{s} \quad (26)$$

Theorem 2 (Bialas Theorem) verifies stability and it can be said that the closed-loop polytopic system with robust controller is stable and the achieved degree of stability α in 4 vertices is 0,0258. Proposed robust PID controller (26) was set on the real process (first tank). Step responses in all three working points are depicted in Fig. 2,3,4.

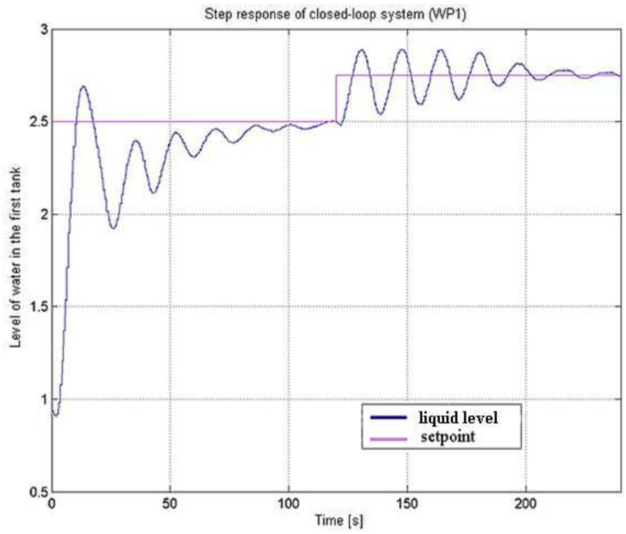

Fig. 2. Step response of closed-loop system in WP1 (the first working point)

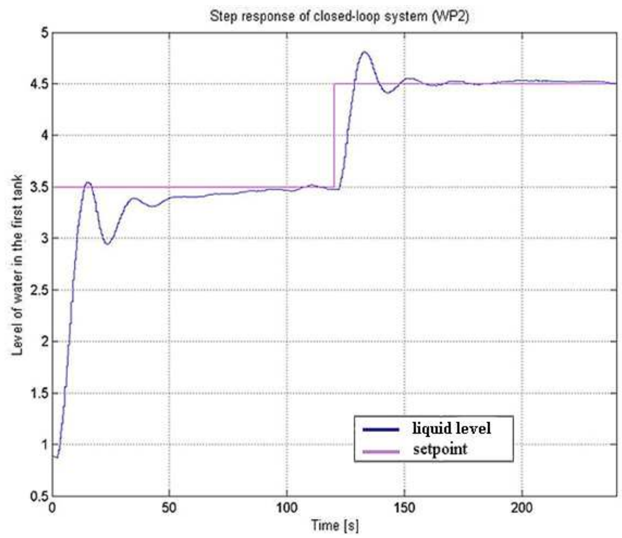

Fig. 3. Step response of closed-loop system in WP2 (the second working point)

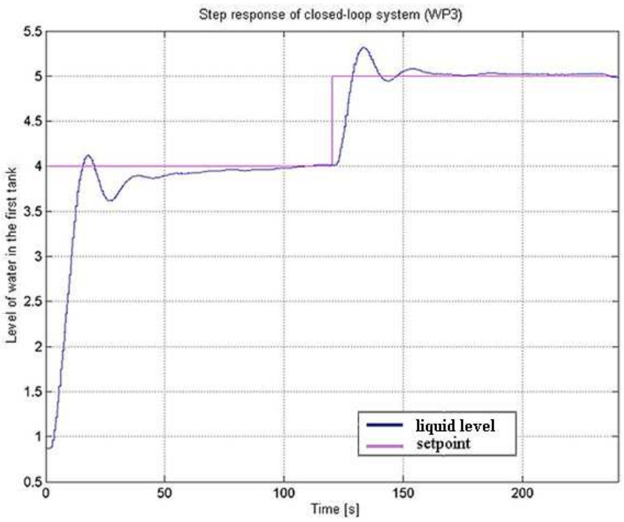

Fig. 4. Step response of closed-loop system in WP3 (the third working point)

4.2 Robust controller design with desired phase margin

Consider the transfer function of nominal model as in (24). Required phase margin φ_R is 45°. In the first step D-curve for parameters K_{P1} and K_D (PD controller) is plotted. These parameters are chosen from stable region above magenta line (see Fig. 5) because it is necessary to stabilize the system (the parameters need not be chosen only from the blue line in this step).

The PD controller has following coefficients: $K_{P1} = 3{,}569$ and $K_D = 0{,}6849$. Poles of characteristic equation of closed-loop system with PD controller are $-0{,}1332 \pm 0{,}2981\mathrm{j}$.

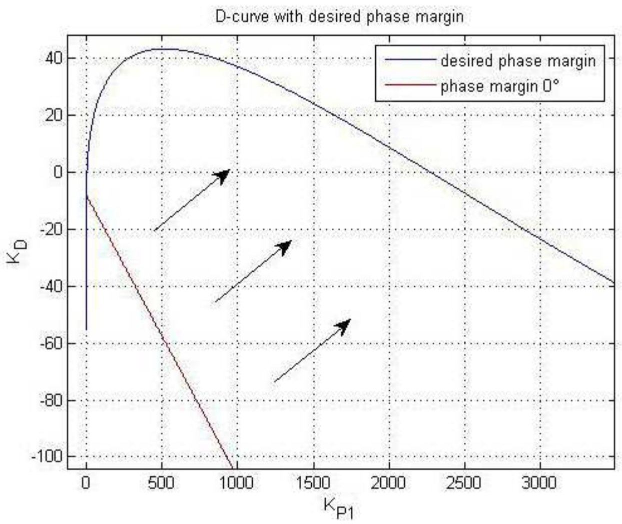

Fig. 5. D-curve for parameters K_{P1} and K_D

The next step consists in design of PI controller for nominal model with PD controller. Parameters of PI controller need to be chosen from plotted desired phase margin 45° (the blue line in Fig. 6). Chosen coefficients of PI controller are $K_{P2} = 0{,}3542$ and $K_I = 0{,}01713$. Transfer function of the final PID controller is

$$G_R(s) = \frac{0{,}2426s^2 + 1{,}276s + 0{,}06114}{s} \tag{27}$$

Poles of characteristic equation of closed-loop system with PID controller (27) are $-0{,}0914 \pm 0{,}1398\mathrm{j}$ and $-0{,}0643$.

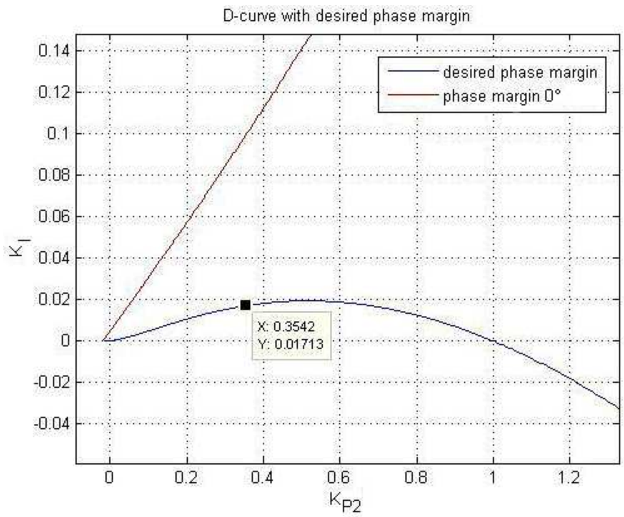

Fig. 6. D-curve for parameters K_{P2} and K_I

Fig. 7 and 8 show that the desired phase margin and robust stability are satisfied. Proposed PID controller (27) was set on the real process. Step responses in all three working points are plotted in Fig. 9, 10 and 11.

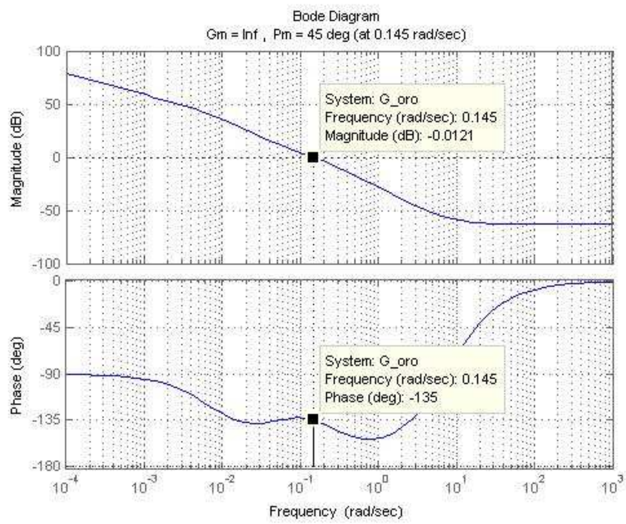

Fig. 7. Bode characteristics for Coupled-Tank process

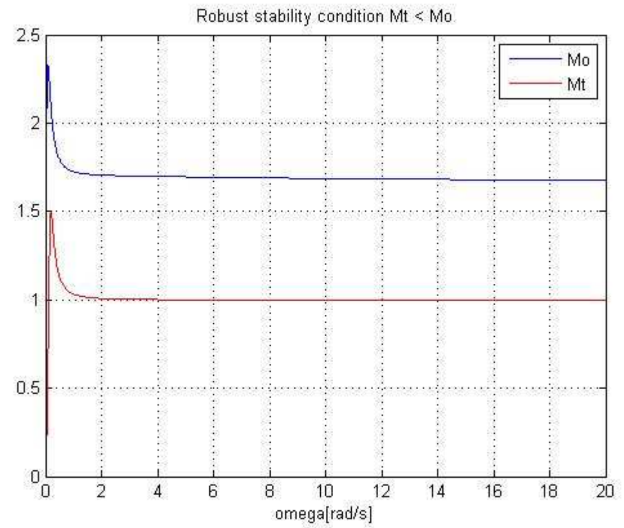

Fig. 8. Robust stability condition

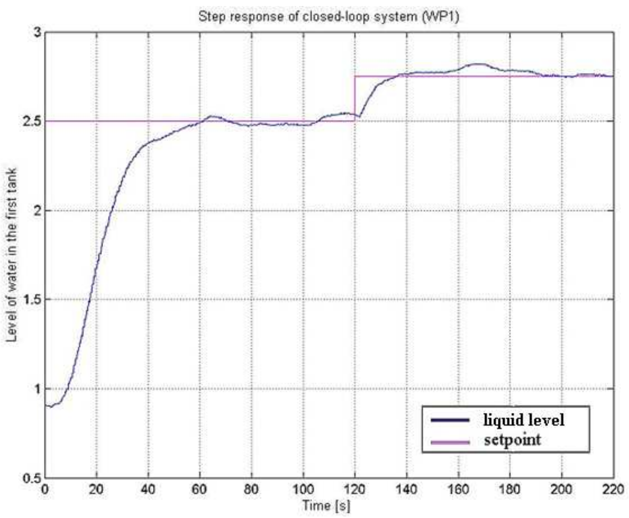

Fig. 9. Step response of closed-loop system in WP1

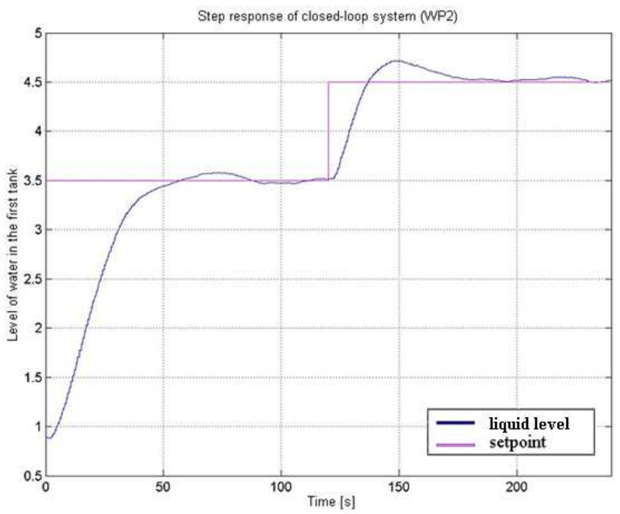

Fig. 10. Step response of closed-loop system in WP2

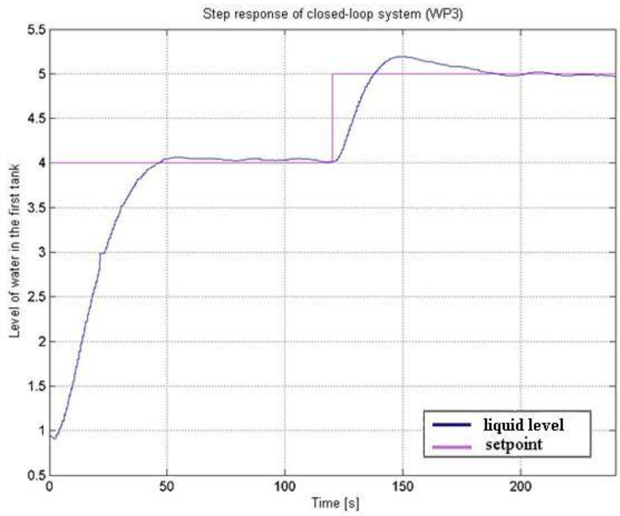

Fig. 11. Step response of closed-loop system in WP3

5. CONCLUSION

In this paper two approaches of robust PID controller design for real unstable Coupled-Tank process have been presented. The first one is Edge Theorem and the second approach is based on modification of the Neimark's D-partition method, which ensures not only stability of closed-loop system but also desired phase margin. From view of control quality, the robust controller design with desired phase margin using Neimark's D-curves is better. Results obtained in the paper will be used for control education at Institute of Control and Industrial Informatics. Further aspects of the studied approach concerning robust controller design, closed-loop or open loop identification of Coupled-Tank process with cascade controller, are under research.

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