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H∞ Controler Design for Active Suspension System

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Abstract: This paper presents the *H∞* synthesis of control for an active suspension design based on an extended quarter-car model. The usage of automobile active suspension has two main reasons, to increase ride comfort and to improve handling performance. Both this requirements are contradictory. To obtain the model performances and solve the *H∞* synthesis the Matlab software with the Robust Control Toolbox has been used. The benefits of controlled active suspensions compared to passive ones are here emphasized.

1. INTRODUCTION

Vehicle suspension has been a hot research topic for many years due to its important role in ride comfort, vehicle safety, road damage minimization and the overall vehicle performances. To meet these requirements, many types of suspension systems, ranging from passive, semi/active, to active suspensions, are currently being employed and studied. It has been well recognized that active suspension has a great potential to meet the tight performance requirements demanded by users. Therefore, in recent years more and more attention has been devoted to the development of active suspensions and various approaches have been proposed to solve the crucial problem of designing a suitable control law for these active suspension systems. In many control applications, it is expected that the behaviour of the designed system will be insensitive (robust) to external disturbance and parameter variations. It is known that feedback in conventional control system has the inherent ability of reducing the effects of external disturbances and parameter variations. In this paper, the H∞ control design problem is converted into a convex optimization problem described by linear matrix inequalities LMI, Zhou (1998).

The *H∞* method addresses a wide range of the control problems, combining the frequency and time-domain approaches. The design is an optimal one in the sense of minimization of the *H∞* norm of the closed-loop transfer function. The *H∞* model includes coloured measurement and process noise. It also addresses the issues of robustness due to model uncertainties, and is applicable to the SISO system as well as to the MIMO system, Gawrovski (2004). In this paper is present the *H∞* control design for quarter-car active suspension system.

2. THE SUSPENSION MODEL

The usually used quarter-car model has two degrees-of-freedom see Fig.1. It includes the vertical motion of the sprung mass m_2 which represents the car body with passengers and the unsprung mass m_1 which corresponds to the mass of the wheel and suspension. The disturbance input w is the road profile. x_1 represent the positions of the sprung mass and x_2 the positions of the unsprung mass.

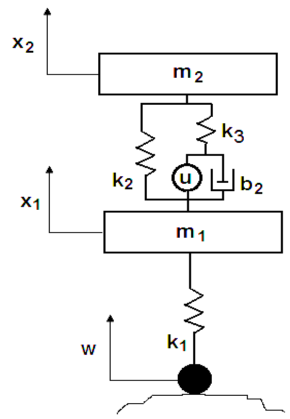

Fig. 1. Extended quarter car with active suspension

Table 1: The values of Parameters in quarter-car

Description	Units	Values
Body (sprung) Mass	m_2 (kg)	350
Axle (unsprung) Mass	m_1 (kg)	35
Suspension Stiffness	k_1 (N/m)	200 000
Suspension Stiffness	k_2 (N/m)	14 000
Tire Damping	b_1 (Ns/m)	500
Tire Damping	b_2 (Ns/m)	1600
Damper Stiffness	k_3 (N/m)	250 000

2.1 Rheological damper model

Usually the suspension is modelled by means of a linear damper and a spring. However also the real spring has basically a linear characteristic, the real damper has a nonlinear and a considerable hysteresis caused primarily by the oil compressibility (bulk modulus – β=0,8 (Pa)). These properties can by well modelled by means of the Maxwell element Fig.2., Guglielmino (2004).

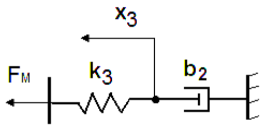

Fig. 2. Rheological damper model (Maxwell element)

The spring k_3 represents the mentioned stiffness of the dampers hydraulics circuit and can by calculated as

$$k_3 = \frac{\beta \cdot S_p^2}{V} = \frac{\beta \cdot \pi^2 d_p^4}{16\ V} \qquad (1)$$

where d_p = 0,022 (m) is the diameter damper rod and $V \approx$ 0,0003 (m^3) is the mean volume of the damper pressure and expanse chambers.

The rheological damper properties for different damper values b_2 are shown in Fig 3.

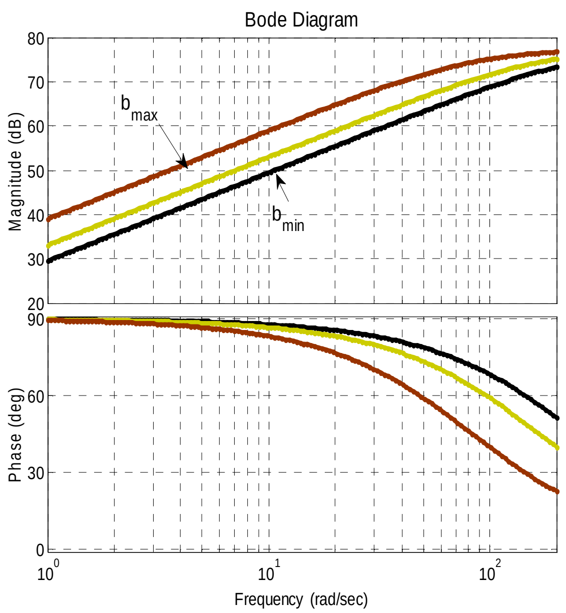

Fig. 3. Characteristic of a rheological damper model

2.2 State space modeling

The state space representation of the controlled system of an extended quarter car model Fig.3 can be formalized as following:

$$\begin{aligned} \dot{x} &= Ax + B_1 w + B_2 u \\ y &= C_1 x + D_{11} w + D_{12} u \\ z &= C_2 x + D_{21} w + D_{22} u \end{aligned} \qquad (2)$$

where the state vector x, output vector y and vector of measurement z are defined as following:

$$x = [x_1 - w_1 \quad x_2 - x_1 \quad \dot{x}_1 \quad \dot{x}_2 \quad \dot{x}_3]^T \qquad (3)$$

$$y = [\ddot{x}_2 \quad x_2 - x_1 \quad F_{dyn}]^T \qquad (4)$$

$$z = [\dot{x}_1 \quad \dot{x}_2 \quad x_2 - x_1]^T \qquad (5)$$

The state space matrices are defined following:

$$A = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 1 & 0 \\ -\frac{k_1}{m_1} & \frac{k_2}{m_1} & 0 & 0 & -\frac{k_3}{m_1} \\ 0 & -\frac{k_2}{m_2} & 0 & 0 & \frac{k_3}{m_2} \\ 0 & 0 & 1 & -1 & -\frac{k_3}{b_2} \end{bmatrix}, \qquad (6)$$

$$B_1 = \begin{bmatrix} -1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \quad B_2 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ -\frac{1}{b_2} \end{bmatrix}, \qquad (7)$$

$$C_1 = \begin{bmatrix} 0 & -\frac{k_2}{m_2} & 0 & 0 & \frac{k_3}{m_2} \\ 0 & 1 & 0 & 0 & 0 \\ \frac{k_1}{m_1} & 0 & 0 & 0 & 0 \end{bmatrix}, \qquad (8)$$

$$D_{11} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \quad D_{12} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \qquad (9)$$

$$C_2 = \begin{bmatrix} -\frac{k_1}{m_1} & \frac{k_2}{m_1} & 0 & 0 & -\frac{k_3}{m_1} \\ 0 & -\frac{k_2}{m_2} & 0 & 0 & \frac{k_3}{m_2} \\ 0 & 1 & 0 & 0 & 0 \end{bmatrix}, \qquad (10)$$

$$D_{21} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \quad D_{22} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}. \qquad (11)$$

2.3 Suspension performances and weighting filters

In this paper, the following performance aspects of quarter-car suspension system are taken into account:

1. Ride comfort – can be quantified by the car body acceleration $\ddot{x}_2$

2. Suspension deflection limitation – the travel space does not need to be minimal but its peak value need to be constrained.

$$|x_2(t) - x_1(t)| \le x_c \tag{12}$$

3. Road holding ability – in order to ensure a firm uninterrupted contract of wheels to road, the dynamic tyre.

The feedback structure is shown in Fig. 4. It includes the input W_1 and output W_2 weighting functions, the extended quarter car model *P(s)* and the controller model *K(s)*.

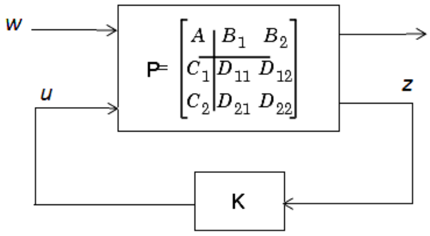

Fig. 4. The active suspension control scheme

The input weight (13) includes the road disturbance filter and the weight for the actuator force.

$$W_1 = \begin{bmatrix} W_{road}(s) & 0 \\ 0 & 1/u_c \end{bmatrix}, \tag{13}$$

$$W_{road}(s) = A_w \, W_{Butter\,0,5Hz}(s) \tag{14}$$

Where the constants A_w represents the power of chosen road type and the $W_{Butter0,5Hz}$ represents a classics high pass Butterworth analogue filter with a cut-off frequency 0,5(Hz). u_c represents the value of the critical force produced by the controlled actuator.

The output weights (14) for the optimized values *y* and for the measured values z are: the matrix of weighting functions is chosen as:

$$W_2 = \begin{bmatrix} W_y & 0 \\ 0 & W_z \end{bmatrix}, \tag{15}$$

$$W_y = \begin{bmatrix} W_{acc}/a_c & 0 & 0 \\ 0 & 1/x_c & 0 \\ 0 & 0 & 1/F_{Dc} \end{bmatrix}, \quad W_z = I \tag{16}$$

where W_{acc} is the weighting filter of acceleration, definite in norm ISO 2631.

$$W_{acc}(s) = \frac{num_w}{den_w} \tag{17}$$

$$num_w = [87{,}72 \quad 1138 \quad 11336 \quad 5453 \quad 5509]$$
$$den_w = [1 \quad 92{,}6854 \quad 2549{,}83 \quad 25969 \quad 81057 \quad 79783]$$

Where the values represent: a_c – critical weighted acceleration acting on the human body chosen from the ISO 2631, x_c – critical suspension deflection given by the suspension design and F_{Dc} – critical dynamic tyre force gravity of the static weight which is acting on the tyre. Dividing each of the optimized parameters with his critical value, we are used normalization and so the weighted and normalized optimized parameters will have no units.

The magnitude frequency characteristics of the road and sprung mass acceleration filters are shown in Fig.5.

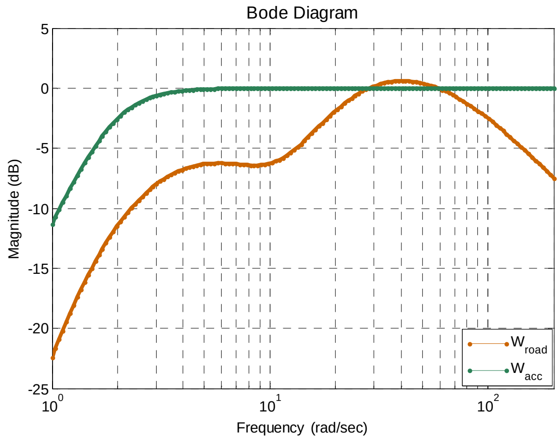

Fig. 5. Bode plot of W_{road} and W_{acc} filters

3. *H∞* CONTROLLER DESING

When open loop is denoted T_{yw}, then a standard optimal *H∞* controller problem is to find admissible controller *K* such that $\|T_{yu}\|_\infty$ is minimal. The problem of finding a suboptimal *H∞* controller can be formulated: for given γ > 0 find all admissible controllers *K*, they exits, such that

$$\|T_{yu}\|_\infty < \gamma \tag{18}$$

3.1 Solution

The solution of this problem requires the solving of two Ricatti equations, one for controller and one for the observer, Gawrovski (2004).

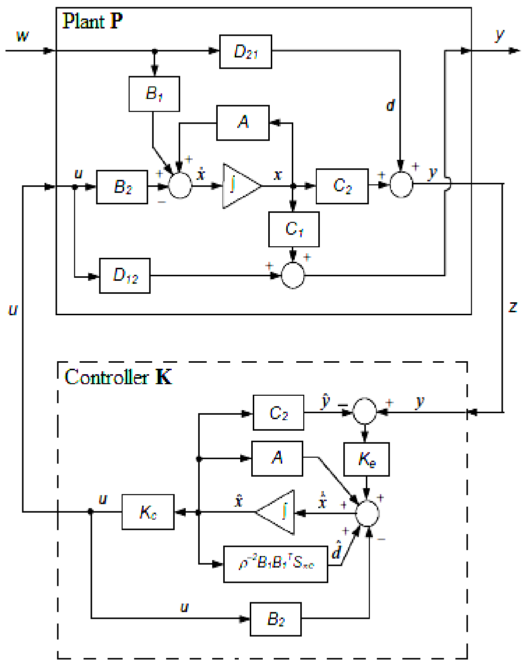

Fig. 6. The central $H\infty$ closed-loop system

The control law is given by

$$u = -K_c \hat{x} \tag{19}$$

and the state estimator equation by

$$\dot{\hat{x}} = Ax + B_2 u + B_1 \hat{w} + K_e (y - \hat{y}) \tag{20}$$

where

$$\hat{w} = \gamma^{-2} B_1^T X_\infty \hat{x} \tag{21}$$
$$\hat{y} = C_2 \hat{x} \tag{22}$$

The controller gain is K_c as for the LQG case, and the estimator gain is $Z_\infty K_e$ instead of K_e as for the LQG case, with

$$K_c = B_2^T X_\infty \tag{23}$$
$$K_e = Z_\infty Y_\infty C_2^T \tag{24}$$
$$Z_\infty = (I - \gamma^{-2} Y_\infty X_\infty)^{-1} \tag{25}$$

The terms X_∞ and Y_∞ are solutions to controller and estimator Ricatti equations

$$X_\infty = Ric \begin{bmatrix} A & \gamma^{-2} B_1 B_1^T - B_2 B_2^T \\ -C_1^T C_1 & -A^T \end{bmatrix} \tag{26}$$

$$Y_\infty = Ric \begin{bmatrix} A^T & \gamma^{-2} C_1^T C_1 - C_2^T C_2 \\ -B_1 B_1^T & -A \end{bmatrix} \tag{27}$$

We do not carry out these calculations by hand – the tools supplied by Matlab *Robust Control Toolbox* just do that.

4. SIMULATION RESULTS

In the next chapter are results in frequency and time domain compared. The results have bean solved using the model shown in the Fig. 1 and its parameters are stated in the Tab. 1.

4.1 Frequency Response Simulations

In the next figures the performance magnitudes of the considered active and passive vehicle suspensions are compared. In Fig.7 the weighted acceleration of the car body is shown. We can read that the active suspension system has better comfort performances from all around the first system eigenfrequency. After the second eigenfrequency the performance of the passive system is better but that is not so important region of frequencies for the comfort criterion an also in real model it is very difficult to control vibrations at so high fervencies. So the active system would acting anyway like a passive one.

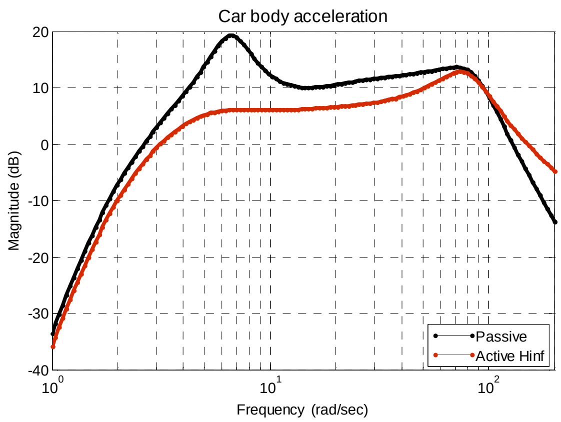

Fig. 7. Bode plot of the weighted and normalized car body vertical acceleration – Comfort criterion

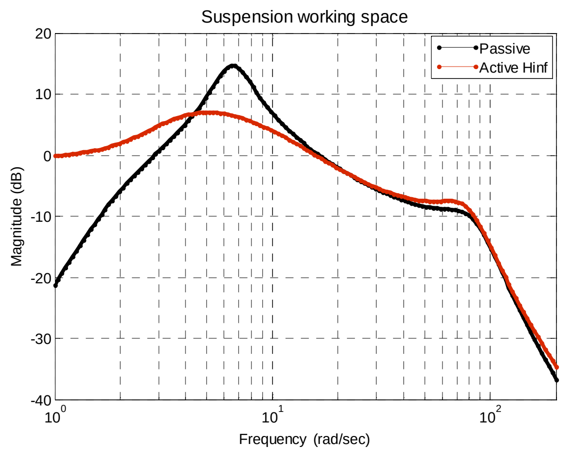

Fig. 8. Bode plot of the normalized suspension deflection – Reliability criterion

In the Fig. 8 the frequency response of the suspension deflection is shown. However there is an increase of the deflection magnitude on the active suspension according to the magnitude of the passive suspension, but at this criterion the most important thing is the maximal value in the whole region of the frequencies - H_∞ norm and this criterion is significantly better achieved with the active suspension.

At the last frequency response Fig. 9 is magnitude of the normalized dynamic tyre force. Here we can see that a significantly improvement by means of the active suspension was achieved and that from all at the first system eigenfrequency.

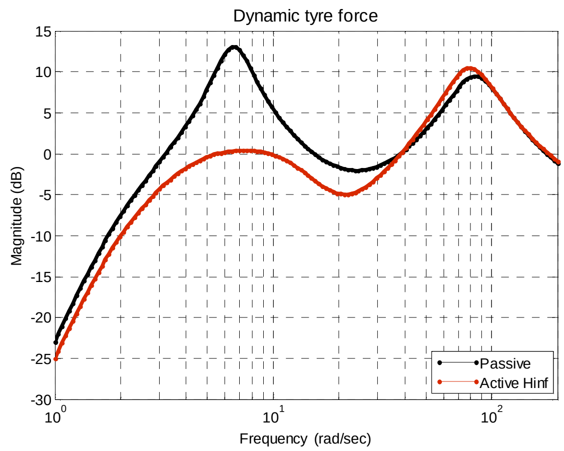

Fig. 9. Bode plot of the normalized dynamics tyre force – Road holding criterion

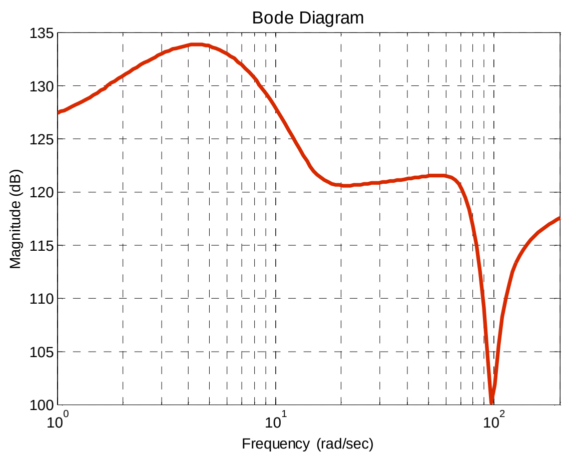

Fig. 10. Bode plot of the actuator

4.2 Time response simulations

Also a time response has been calculated which shows how the passive and active suspension systems are responding by crossing a road bump disturbance see Fig. 11 – 14.

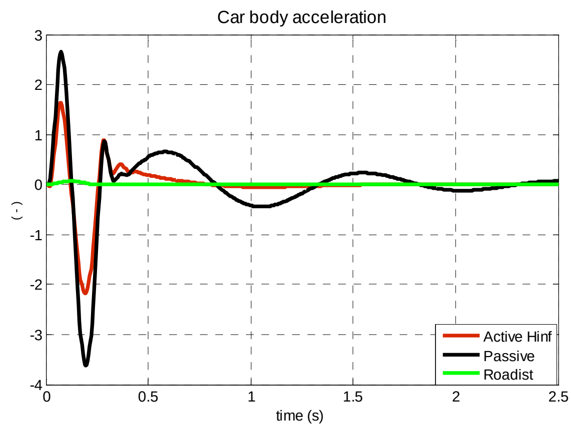

Fig. 11. Time response of the vertical acceleration (active, passive suspension and road disturbance)

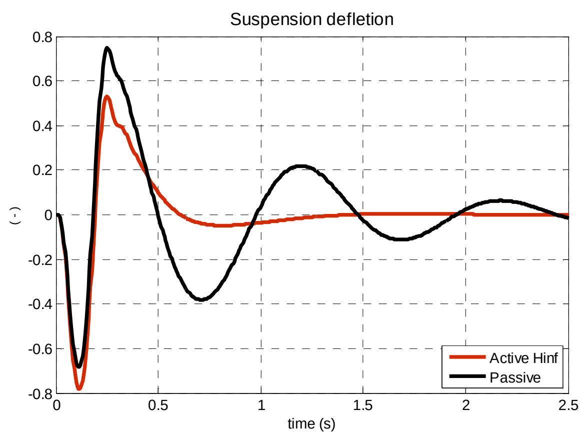

Fig. 12. Time response of the suspension deflection

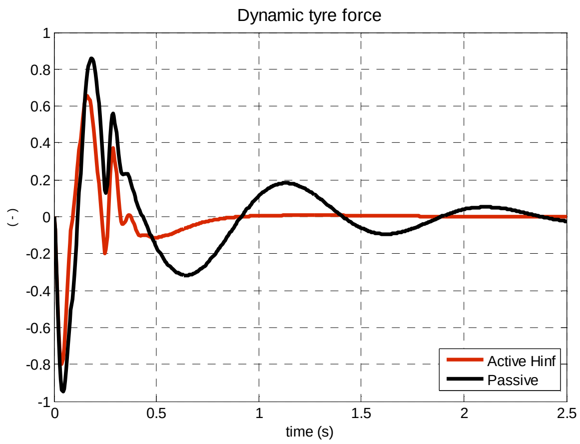

Fig. 13. Time response of dynamic tyre force

In the next table the suspensions performance values calculated via the H_2 and $H\infty$ norms from the previous time responses are shown.

Table 2: Performance values of passive and active suspension

Suspension performances (-)	passive H_2	active H_2	passive $H\infty$	active $H\infty$
Car body acceleration	0.7476	0.4342	3.6275	2.1830
Suspension deflection	0.2509	0.1824	0.7866	0.7434
Dynamic tire force	0.2420	0.1517	0.9494	0.7974

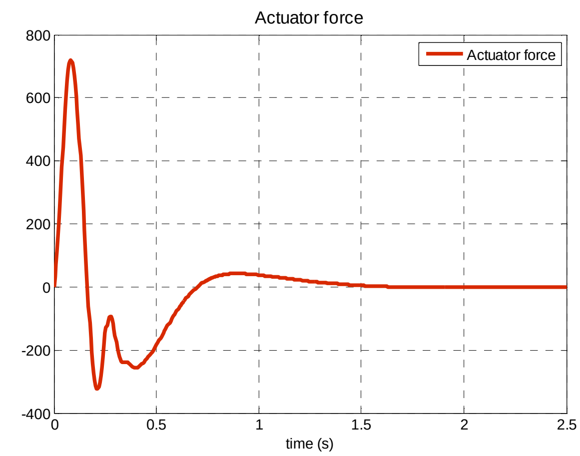

Fig. 14. Time response of actuator force

CONCLUSION

From the simulations results we can clearly see that the active controlled suspension with $H\infty$ controller offers a much better suspension performances as the classics passive suspension model. These results have been confirmed also even if we have extended the simple quarter car model with the damper stiffness which has brought one more degree of freedom into the system and also mead the simulation model more realistic.

ACKNOWLEDGMENTS

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REFERENCES

Wodek K. Gawrovski (2004). *Advanced Structural Dynamics and Aktive Control of Structurec* Springer-Verlag New York, Inc.

Yamashita, M., Fujimori, K., Hayakawa, K., and Kimura, H. (1994). *Application of H-infinity Control to Active Suspension Systems*. *Automatica*, 30(11): 1717-1792.

K. Zhou, J.C.Doyle, and K. Glover (1996). *Robust and Optimal Control*. Prentice-Hall, Upper Saddle River

Zhou, K. and Doyle J.C. (1998). *Essentials of Robust Control* Prentice Hall.

Bruce A. Francis (1987), *Lecture Notes in Control and Information Sciences*, Springer – Verlag Berlin Heidelberg,

P. Gahinet, A. Nemirovski, A. J. Laub, and M. Chilali(1995) *LMI Control Toolbox for Use with MATLAB*. The MatWorks Inc.

Rajamani, R., (2006) *Vehicle Dynamics and Control*, Rensselaer Polytechnic Institute, Troy, NY 12180-3590, USA

Emanuele Guglielmino, Tudor Sireteanu, Charles W. Stammers, Chorghe Ghita, Marius Giuclea, (2004) *Semi-active Suspension Control*, London