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Robust elimination lemma: sufficient condition for robust output feedback controller design

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Abstract: A linear algebra result known as Elimination lemma is frequently used in lot of filtering and control problems to transform products of unknown matrices to LMI form, however, the robust counterpart to elimination lemma is not known. In this paper, sufficient robust stability condition inspired by elimination lemma is developed and the respective robust static output feedback controller design procedure based on LMI formulation and solution is proposed. The proposed robust controller design procedure is computationally not demanding and is illustrated on examples.

1. INTRODUCTION

Linear algebraic result known as Elimination lemma plays an important role in the study of robust stability conditions for linear systems with polytopic uncertainties (Boyd et al. 1994, Skelton et al. 1998). The following matrix inequality often appears in robust control problem formulation

$$G_i + U_i X V_i + V_i^T X^T U_i^T < 0, \qquad i = 1,2,...,N \tag{1}$$

The matrices G_i, U_i, V_i, X may all depend on the control system parameters to be designed; X may represent control gain which is the same for the whole uncertainty domain, when robust controller is concerned. Elimination lemma enables to eliminate unknown matrix X from (1) when N = 1, thus simplifying the resulting design inequality, which then often turns to LMI. Unfortunately, for $N > 1$, which is the case of uncertain polytopic linear system, the elimination lemma cannot be directly extended, (deOliveira 2005; Vesely, et al 2009). Moreover, it is not clear if such counterpart in the form of necessary and sufficient condition can be found since the class of structured linear control problems such as decentralized control and simultaneous static output feedback (SOF) belongs to NP hard problems as have been proven in (Blondel and Tsitsiklis, 1997). Nevertheless, various techniques have been developed to reformulate the problem as LMI one using certain convex approximation as linearizing or convexifying functions (deOliveira et al., 2000; Han and Skelton, 2003, Veselý, 2003; Rosinová and Veselý, 2003). The problem remains in linearizing the off-diagonal terms, since in this case, the upper bound based linearization formulas are not quite suitable to receive workable results.

In this paper, sufficient condition for (1) is developed, which can be advantageously used for robust static output feedback controller design. The respective control design procedure, which is computationally not demanding, is presented. Section 2 brings problem formulation and preliminaries. In Section 3, the sufficient condition for (1) is developed, in which the unknown matrix X is eliminated from off-line terms of the respective matrix determining robust stability condition. The corresponding robust control design procedure is proposed and in Section 4 it is illustrated on two examples.

2. PRELIMINARIES AND PROBLEM FORMULATION

The robust static output feedback control design problem is formulated in this section and the respective sufficient robust stability condition in the form (1) is presented. Consider the class of linear uncertain continuous or, alternatively, discrete-time systems described by convex polytopic model:

$$\begin{aligned} \delta x(t) &= A(\alpha)x(t) + B(\alpha)u(t) \\ y(t) &= Cx(t) \end{aligned} \tag{2}$$

where

$\delta x(t) = \dot{x}(t)$ for continuous - time system
$\delta x(t) = x(t+1)$ for discrete - time system

$x(t) \in R^n, u(t) \in R^m, y(t) \in R^l$ are state, control and output vectors respectively; uncertain model matrices $A(\alpha), B(\alpha)$ are from convex polytopic uncertainty domain given by polytope vertices $A_i \in R^{nxn}, B_i \in R^{nxm}$, $i = 1,..., N$:

$(A(\alpha), B(\alpha)) \in S,$

$$S = \left\{ (A(\alpha), B(\alpha)) : \; A(\alpha) = \sum_{i=1}^{N} \alpha_i A_i, B(\alpha) = \sum_{i=1}^{N} \alpha_i B_i, \sum_{i=1}^{N} \alpha_i = 1, \; \alpha_i \geq 0 \right\} \tag{3}$$

Consider a static output feedback control law

$$u(t) = Fy(t) = FCx(t) \tag{4}$$

and the respective closed loop uncertain system

$$\delta x(t) = A_C(\alpha)x(t) \tag{5}$$

where

$$A_C(\alpha) \in \left\{ \sum_{i=1}^{N} \alpha_i A_{Ci}, \quad \sum_{i=1}^{N} \alpha_i = 1, \quad \alpha_i \geq 0 \right\}. \tag{6}$$

$A_{Ci} = A_i + B_i FC$

To study the stability of uncertain linear system (2), the parameter-dependent quadratic Lyapunov function is used

$$V(t) = x(t)^T P(\alpha) x(t) \tag{7}$$

and the respective robust stability condition is considered in compliance with (Oliveira et al. 1999).

Definition 1

System (5) is *robustly stable* in the convex uncertainty domain (6) with parameter-dependent Lyapunov function (7) if and only if there exists a matrix $P(\alpha) = P(\alpha)^T > 0$ such that

$$\begin{aligned} r_{12} P(\alpha) A_C(\alpha) + r_{12}^* A_C^T(\alpha) P(\alpha) + r_{11} P(\alpha) + \\ + r_{22} A_C^T(\alpha) P(\alpha) A_C(\alpha) < 0 \end{aligned} \tag{8}$$

for all α such that $A_C(\alpha)$ is given by (6) and r_{11}= 0, r_{12}= 1, r_{22}= 0 for a continuous-time system; r_{11}= -1, r_{12}= 0, r_{22}= 1 for a discrete-time system.

In the following we consider Lyapunov matrix $P(\alpha)$ in the form

$$P(\alpha) = \sum_{i=1}^{N} \alpha_i P_i \quad \text{where } P_i = P_i^T > 0, \; j = 1,...,N \tag{9}$$

Robust static output feedback control design aims at finding an output feedback gain matrix *F* for control law (4) so that the uncertain closed loop system (5) is robustly stable.

Recall a sufficient robust stability condition proposed in (Peaucelle et al., 2000), which has been favoured in comparison of several available results (Grman et al. 2005)

Lemma 1

If there exist matrices $E \in R^{nxn}, H \in R^{nxn}$ and *N* symmetric positive definite matrices $P_i \in R^{nxn}$ such that for all *i*=1,…,*N*:

$$\begin{bmatrix} r_{11} P_i + A_{Ci}^T E^T + E A_{Ci} & r_{12} P_i - E + A_{Ci}^T H \\ r_{12}^* P_i - E^T + H^T A_{Ci} & r_{22} P_i - (H + H^T) \end{bmatrix} < 0 \tag{10}$$

$A_{Ci} = A_i + B_i FC$

then system (5) is robustly stable, where r_{11}= 0, r_{12}= 1, r_{22}= 0 for a continuous-time system; r_{11}= -1, r_{12}= 0, r_{22}= 1 for a discrete-time system.

Matrix inequality (10) is in the form of LMI for robust stability analysis with for unknown matrices E, H, P_i. On the contrary, for robust control design (10) is no more LMI since in this case, *F* is unknown matrix as well as E, H, P_i, and these unknown matrices appear in bilinear terms. One possibility to cope with nonlinear (bilinear) terms is to use bilinear matrix inequality (BMI) solvers; this approach, however, has its limitations (e.g dimension of problem, case sensitivity). To improve numerical tractability, there is an effort to transform (10) to LMI, the frequent approach is to employ linearization (deOliveira et al. 2000). In this paper the upper bound on the left hand side of (10) is used, based on the following well known matrix inequality.

Lemma 2

For any $\varepsilon_i > 0$ following inequalities hold for any matrices U_i, V_i, X

$$\begin{aligned} U_i X V_i + V_i^T X^T U_i^T \leq \varepsilon_i^{-1} U_i U_i^T + \varepsilon_i V_i^T X^T X V_i, \\ i = 1,2,...,N \end{aligned} \tag{11}$$

Lemma 2 immediately follows from inequality

$$\left(\frac{U_i^T}{\sqrt{\varepsilon_i}} - \sqrt{\varepsilon_i} X V_i \right)^T \left(\frac{U_i^T}{\sqrt{\varepsilon_i}} - \sqrt{\varepsilon_i} X V_i \right) \geq 0, \qquad i = 1,2,...,N \tag{12}$$

which holds for any $\varepsilon_i > 0$ and any matrices U_i, V_i, X.

A closed loop performance is assessed considering the *guaranteed cost* notion. The quadratic cost function is used.

$$J_c = \int_0^\infty [x(t)^T Q x(t) + u(t)^T R u(t)] dt$$

for a continuous-time and

$$J_d = \sum_{k=0}^{\infty} [x(t)^T Q x(t) + u(t)^T R u(t)]$$

for a discrete-time system, (13)

where $Q \in R^{n \times n}, R \in R^{m \times m}$ are symmetric positive definite matrices.

Control law (4) is called guaranteed cost control when there exist a feedback gain matrix *F* and a constant J_0 such that

$$J \leq J_0 \tag{14}$$

holds for the closed loop system (5), (6); J_0 is the guaranteed cost.

Extending robust stability condition (10) by guaranteed cost requirement as known from LQ theory, the robust stability condition with guaranteed cost is obtained in the form

$$\begin{bmatrix} r_{11} P_i + A_{Ci}^T E^T + E A_{Ci} + Q + C^T F^T R F C & r_{12} P_i - E + A_{Ci}^T H \\ r_{12}^* P_i - E^T + H^T A_{Ci} & r_{22} P_i - (H + H^T) \end{bmatrix} < 0 \tag{15}$$

Inequality (15) is LMI for stability analysis, i.e. for unknown matrices P_i, E, H. In the case of controller design, where *F* is also unknown, the bilinear terms appears in (15) both in diagonal and off-diagonal terms. Nonlinear diagonal terms in (15) can be treated by existing convexifying approaches as in (deOliveira et al., 2000; Han and Skelton, 2003, Veselý et al, 2009).

The respective potential convexifying function for terms as X^{-1} and XWX has been proposed in the linearizing form (Han and Skelton, 2003):

- The linearization of $X^{-1} \in R^{nxn}$ about the value $X_k > 0$ is

$$\Phi(X^{-1}, X_k) = X_k^{-1} - X_k^{-1}(X - X_k)X_k^{-1} \qquad (16)$$

- The linearization of $XWX \in R^{nxn}$ about X_k is

$$\Psi(XWX, X_k) = -X_k W X_k + XWX_k + X_k WX \;. \qquad (17)$$

Both functions defined in (16) and (17) meet one of the basic requirements on convexifying function: to be equal to the original nonconvex term if and only if $X_k = X$. However, the question how to choose the appropriate "nice" convexifying function remains still open.

In fact, linearization (16) and (17) is based on using upper bounds on bilinear terms, which is suitable for treating diagonal terms in (15). As soon as the bilinear terms in the off-diagonal part of testing matrix are to be linearized, the upper bounds based approaches are no more appropriate and a different way to linearization must be found. One possible way to transform robust stability condition to LMI is proposed in the next section.

In the sequel, $X > 0$ denotes positive definite matrix; * in matrices denotes the respective transposed term to make the matrix symmetric; I denotes identity matrix and 0 denotes zero matrix of the respective dimensions.

3. ROBUST SOF CONTROLLER DESIGN PROCEDURE

In this section, the novel static output feedback design procedure is proposed based on sufficient robust stability condition, which yields LMI formulation for controller design.

The following corollary of Lemma 2 provides the bound (sufficient condition) which will be used below.

Corollary 1

If there exists $\varepsilon_i > 0$ such that

$$G_i + \varepsilon_i^{-1} U_i U_i^T + \varepsilon_i V_i^T X^T X V_i < 0, \quad i = 1,2,...,N \qquad (18)$$

then

$$G_i + U_i X V_i + V_i^T X^T U_i^T < 0, \qquad i = 1,2,...,N \qquad (19)$$

Sufficient robust stability condition is then formulated in Theorem 1.

Theorem 1

If there exist matrices $E \in R^{nxn}, H_1 \in R^{nxn}, H_2 \in R^{nxn}$, N symmetric positive definite matrices $P_i \in R^{nxn}$ and $\varepsilon_i > 0$ such that for all $i = 1,\ldots,N$:

$$\widetilde{G}_i \leq 0 \qquad (20)$$

where $\widetilde{G}_i = \begin{bmatrix} g_{i11} & g_{i12} \\ g_{i21} & g_{i22} \end{bmatrix}$

$$g_{i11} = r_{11}P_i + A_{Ci}^T E^T + EA_{Ci} + Q + C^T F^T RFC + \varepsilon_i^{-1} A_{Ci}^T A_{Ci}$$
$$g_{i12} = r_{12}P_i - E + A_{Ci}^T H_1 - \varepsilon_i^{-1} A_{Ci}^T$$
$$g_{i21} = r_{12}^* P_i - E^T + H_1^T A_{Ci} - \varepsilon_i^{-1} A_{Ci}$$
$$g_{i22} = r_{22}P_i - (H_1 + H_1^T) + \left(\rho_X^2 \varepsilon_i + \varepsilon_i^{-1}\right) I$$

and ρ_X is a chosen positive constant,

then closed loop uncertain system (5), (6) is stable with guaranteed cost.

Proof

Robust stability condition (10) can be for $H = H_1 + H_2$ rewritten in the form (19) where

$$G_i = \begin{bmatrix} r_{11}P_i + A_{Ci}^T E^T + EA_{Ci} + Q + C^T F^T RFC & r_{12}P_i - E + A_{Ci}^T H_1 \\ r_{12}^* P_i - E^T + H_1^T A_{Ci} & r_{22}P_i - (H_1 + H_1^T) \end{bmatrix}$$

$$U_i^T = \begin{bmatrix} A_{Ci} & -I \end{bmatrix}, \; X = H_2, \;\; V_1 = \begin{bmatrix} 0 & I \end{bmatrix}. \qquad (21)$$

Let us apply now Corollary 1 to the matrices defined in (21).

$$U_i U_i^T = \begin{bmatrix} A_{Ci}^T \\ -I \end{bmatrix} \begin{bmatrix} A_{Ci} & -I \end{bmatrix} = \begin{bmatrix} A_{Ci}^T A_{Ci} & -A_{Ci}^T \\ -A_{Ci} & I \end{bmatrix}$$
$$V_i^T H_2^T H_2 V_i = \begin{bmatrix} 0 \\ I \end{bmatrix} H_2^T H_2 \begin{bmatrix} 0 & I \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & H_2^T H_2 \end{bmatrix} \qquad (22)$$

H_2 is any matrix, let us consider it as diagonal: $H_2 = \rho_X I$, where ρ_X is a chosen constant.

Substituting (21) and (22) into (18), the sufficient robust stability condition (20) is obtained. □

Note, that the nonlinear (bilinear) terms appear only in the diagonal part of matrix $\widetilde{G}_i$ in robust stability condition (20).

The robust SOF controller can be now designed using (20) as described above, by the following proposed procedure.

Procedure for robust SOF controller design:

1. Choose an upper bound constant r_0 for

$$0 \leq P_i \leq r_0 I$$

2. Choose $H_1 \approx 0.8 r_0$ and $\rho_X \approx 0.04 r_0$.

3. Choose starting value of $\varepsilon_i > 0$

4. Apply linearization of the diagonal terms using (17) and the respective iterative procedure and solve LMI (20) for unknown matrices P_i, E, H, F.

If (20) is infeasible, change the constant $\varepsilon_i > 0$ and repeat steps 3. and 4.

The outlined procedure requires iterative computation in steps 3. and 4., in fact it is one dimensional search for appropriate value of $\varepsilon_i > 0$ so that the outlined procedure provides feasible solution of (20).

4. EXAMPLES

In this section the proposed robust controller design procedure is illustrated on two examples. The previous result for robust SOF design with guaranteed cost (Veselý et al, 2009) is recalled and used for comparison.

Sufficient robust stability condition for uncertain system (5), (6) with guaranteed cost can be formulated in the following form (Veselý et al, 2009):

If there exist matrices $E \in R^{nxn}, H \in R^{nxn}$, N symmetric positive definite matrices $P_i \in R^{nxn}$ and $\gamma > 0$ such that for all $i = 1,\ldots,N$:

$$\begin{bmatrix} w_{11} & r_{12}P_i - H & A_{Ci}^T \\ r_{12}P_i - H^T & r_{22}P_i - 2\gamma I + I & 0 \\ A_{Ci} & 0 & -\gamma^2 I \end{bmatrix} \leq 0 \tag{23}$$

where

$$w_{11} = r_{11}P_i + A_{Ci}^T E^T + EA_{Ci} + Q + C^T F^T RFC$$

then closed loop uncertain system (5), (6) is stable with guaranteed cost.

In inequality (23) scalar parameter $\gamma > 0$ is to be appropriately chosen.

Example 1

Consider uncertain system with 10 states, 2 inputs and 4 outputs with nominal model described by matrices A_0, B_0, C and uncertainty matrices $A_{u1}, A_{u2}, B_{u1}, B_{u2}$.

$$A_0 = \begin{bmatrix} 0 & -0.2148 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & -1.0142 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -0.2605 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & -0.9107 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -0.1639 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & -0.8137 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -0.2279 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & -0.8251 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 \end{bmatrix}$$

$$A_{u1} = \begin{bmatrix} 0 & -0.025 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -0.1395 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -0.0938 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -0.2911 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.0188 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.0208 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -0.0333 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -0.1173 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$A_{u2} = \begin{bmatrix} 0 & 0.0125 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.0594 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -0.0938 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -0.2911 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.0188 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.0208 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -0.0333 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -0.1173 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$B_0 = \begin{bmatrix} 0.3148 & 0 \\ 0.0478 & 0 \\ 0 & -0.1028 \\ 0 & -0.0091 \\ -0.0841 & 0 \\ -0.0287 & 0 \\ 0 & 0.3676 \\ 0 & 0.2448 \\ 0 & 0 \\ 0 & 0 \end{bmatrix},$$

$$B_{u1} = \begin{bmatrix} 0.0625 & 0 \\ -0.0798 & 0 \\ 0 & -0.0462 \\ 0 & 0.0449 \\ 0.0016 & 0 \\ 0.0072 & 0 \\ 0 & 0.0770 \\ 0 & -0.0050 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}, B_{u2} = \begin{bmatrix} -0.0094 & 0 \\ 0.0151 & 0 \\ 0 & 0.0019 \\ 0 & -0.003 \\ -0.0121 & 0 \\ -0.03 & 0 \\ 0 & -0.0640 \\ 0 & 0.0189 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$$

The respective uncertain polytopic model (2), (3) for the above matrices has 4 vertices given by:

$$A_i = A_0 + q_1 A_{u1} + q_2 A_{u2}, B_i = B_0 + q_1 B_{u1} + q_2 B_{u2}, \tag{24}$$

where $q_1, q_2 \in \{-1,1\}$.

Output matrix is

$$C = \begin{bmatrix} 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

Cost function matrices are: $Q = 0.1I, R = I$.

The robust controller design procedure proposed in the end of Section 3 has been realized for the above described uncertain system. Upper bound on Lyapunov matrix P_i has been chosen as $r_0 = 1000$, $H_1 = 0.8r_0$ and $\rho_X = 0.04r_0$. Results obtained using new design procedure and results corresponding to a solution of (23) are summarized as follows.

New procedure based on (20):

values of ε providing a feasible solution to (20): $\varepsilon \in \langle 0.006, 0.016 \rangle$

SOF gain matrix for $\varepsilon = 0.0075$:

$$F_1 = \begin{bmatrix} -1.1873 & 0 & -0.1462 & 0 \\ 0 & -0.7996 & 0 & -0.0586 \end{bmatrix}$$

maximal closed-loop system eigenvalue = -0.0397

maximal eigenvalue of Lyapunov matrices = 999,899.

Results for previously designed procedure – solution of (23):

values of γ providing a feasible solution to (23): $\gamma \in \langle 0.51, 4.71 \rangle$

SOF gain matrix for $\gamma = 1.51$:

$$F_2 = \begin{bmatrix} -1.9138 & 0 & -0.2725 & 0 \\ 0 & -1.4172 & 0 & -0.1026 \end{bmatrix}$$

maximal closed-loop system eigenvalue = -0.0496

maximal eigenvalue of Lyapunov matrices = 942,388.

Example 2

Consider double integrator described by a nominal model

$$A_0 = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, B_0 = \begin{bmatrix} 0 \\ 0.75 \end{bmatrix}, C = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

with uncertainties:

$$A_{u1} = \begin{bmatrix} 0 & -0.1 \\ 0 & 0 \end{bmatrix}, A_{u2} = \begin{bmatrix} 0 & 0.1 \\ 0 & 0 \end{bmatrix}, B_{u1} = \begin{bmatrix} 0 \\ -0.25 \end{bmatrix}, B_{u2} = \begin{bmatrix} 0 \\ 0.25 \end{bmatrix}$$

Analogically as in Example 1, uncertain system is described by a polytope with four vertices given by (24).

Cost function matrices are: $Q = 0.1I, R = I$.

Parameters for new design procedure has been the same as in Example 1: Upper bound on Lyapunov matrix P_i chosen as $r_0 = 1000$, $H_1 = 0.8r_0$ and $\rho_X = 0.04r_0$. Results obtained using new design procedure and results corresponding to a solution of (23) are summarized below.

New procedure based on (20):

values of ε providing a feasible solution to (20): $\varepsilon \in \langle 0.001, 0.026 \rangle$

SOF gain matrix for $\varepsilon = 0.0055$:

$$F_1 = \begin{bmatrix} -0.3716 & -1.9369 \end{bmatrix}$$

maximal closed-loop system eigenvalue = -0.2275

maximal eigenvalue of Lyapunov matrices = 866.95.

Results for previously designed procedure – solution of (23):

values of γ providing a feasible solution to (23): $\gamma \in \langle 0.51, 10.51 \rangle$

SOF gain matrix for $\gamma = 2.31$:

$$F_2 = \begin{bmatrix} -22.4311 & -266.0514 \end{bmatrix}$$

maximal closed-loop system eigenvalue = -0.0675

maximal eigenvalue of Lyapunov matrices = 889,52.

In both examples the proposed procedure has been successfully applied to compute the robust control gain matrix including guaranteed cost requirement.

5. CONCLUSION

Robust static output feedback control design procedure has been proposed based on new developed sufficient robust stability condition. This condition is in the form of matrix inequality, where the off diagonal terms of testing matrix are linear with respect to unknown matrices and bilinear terms in the matrix diagonal can be readily linearized using upper bound linearization approach. The proposed procedure includes scalar parameters to be chosen by a designer, the proposed values of these parameters have been tested on various examples, and two of them are shown in Section 4.

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REFERENCES

Blondel, V. and J.N. Tsitsiklis (1997). NP-hardness of some linear control design problems. *SIAM J. Control Optim.*, **35**, pp.2118-2127.

Boyd, S., L. El Ghaoui, E. Feron and V. Balakrishnan (1994). Linear matrix inequalities in system and control theory. *SIAM Studies in Applied mathematics*, Philadelphia.

deOliveira, M.C., J. Bernussou and J.C. Geromel (1999). A new discrete-time robust stability condition. *Systems and Control Letters*, **37**, pp. 261-265.

de Oliveira M.C., J.F. Camino and R.E. Skelton (2000). A convexifying algorithm for the design of structured linear controllers. *In: Proceedings of the 39 IEEE Conf on Decision and Control, Sydney*, 2000, pp. 2781-2786.

de Oliveira M.C. (2005). A robust version of the elimination lemma. In:16th Triennial IFAC World Congress, Prague, 2005, CD-ROM.

Grman L., D. Rosinová, V. Veselý and A. Kozáková (2005). Robust stability conditions for polytopic systems. *Int.J. of Systems Science,* 36 (N15), pp. 961-974.

Henrion D., J. Löfberg, M. Kocvara and M. Stingl (2005). Solving polynomial static output feedback problems with PENBMI. LAAS-CNRS Research Report No. 05165, March 2005. Proceedings of the joint IEEE Conference on Decision and Control and European Control Conference, Sevilla, Spain, December 2005.

Peaucelle, D., D. Arzelier, O. Bachelier and J. Bernussou (2000). A new robust *D*-stability condition for real convex polytopic uncertainty. *Systems and Control Letters*, **40**, pp. 21-30.

Rosinová, D. and V. Veselý (2003). Robust output feedback design of discrete-time systems – linear matrix inequality methods. In: *2th IFAC Conf. CSD'03* (CD-ROM), Bratislava, Slovakia.

Skelton, R.E., T. Iwasaki and K. Grigoriadis (1998). *A Unified Algebraic Approach to Linear Control Design*, Taylor and Francis.

Veselý, V., Rosinová D., and Kučera V. (2009). Robust controller design for polytopic systems via Robust Elimination Lemma. Journal of Electrical Eng. Vol.60, N1, 34-38.