CONTROLLED VARIABLE AND MEASUREMENT SELECTION

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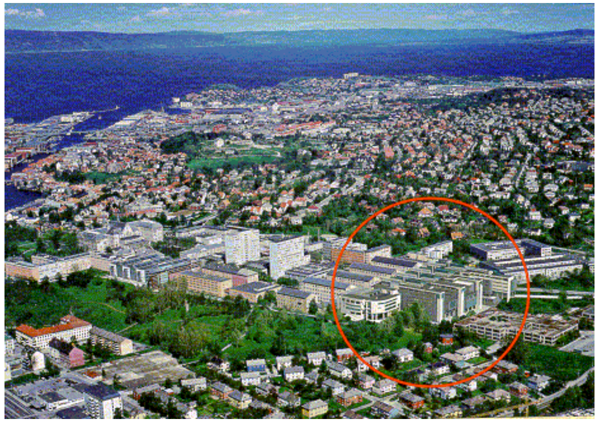

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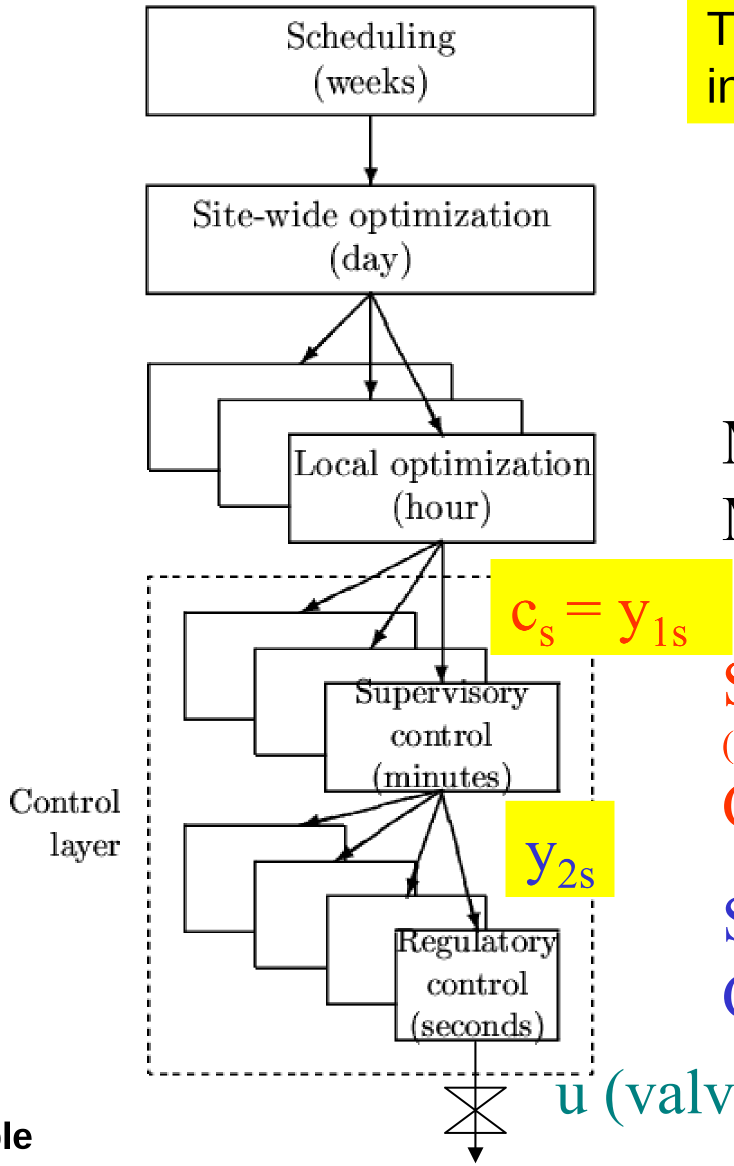
Plantwide control

Process control



RTO

MPC

PID

u (valves)

The controlled variables (CVs) interconnect the layers

OBJECTIVE

Min J (economics); MV=y_{1s}

Switch+Follow path
(+ look after other variables)
CV=y_1 (+ u); MV=y_{2s}

Stabilize + avoid drift
CV=y_2; MV=u

MV = manipulated variable
CV = controlled variable

- Alan Foss ("Critique of chemical process control theory", AIChE Journal,1973):

The central issue to be resolved ... is the determination of control system structure. ***Which variables should be measured, which inputs should be manipulated and which links should be made between the two sets?*** *There is more than a suspicion that the work of a genius is needed here, for without it the control configuration problem will likely remain in a primitive, hazily stated and wholly unmanageable form. The gap is present indeed, but contrary to the views of many, it is the theoretician who must close it.*

This talk:
Controlled variable (CV) selection: What should we measure and control?

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Implementation of optimal operation

- Optimal operation for given d^*:

$$\min_u \; J(u,x,d)$$

subject to:

Model equations: $f(u,x,d) = 0$

Operational constraints: $g(u,x,d) < 0$

$\rightarrow u_{opt}(d^*)$

Problem: Usally cannot keep u_{opt} constant because disturbances d change

How should we adjust the degrees of freedom (u)?

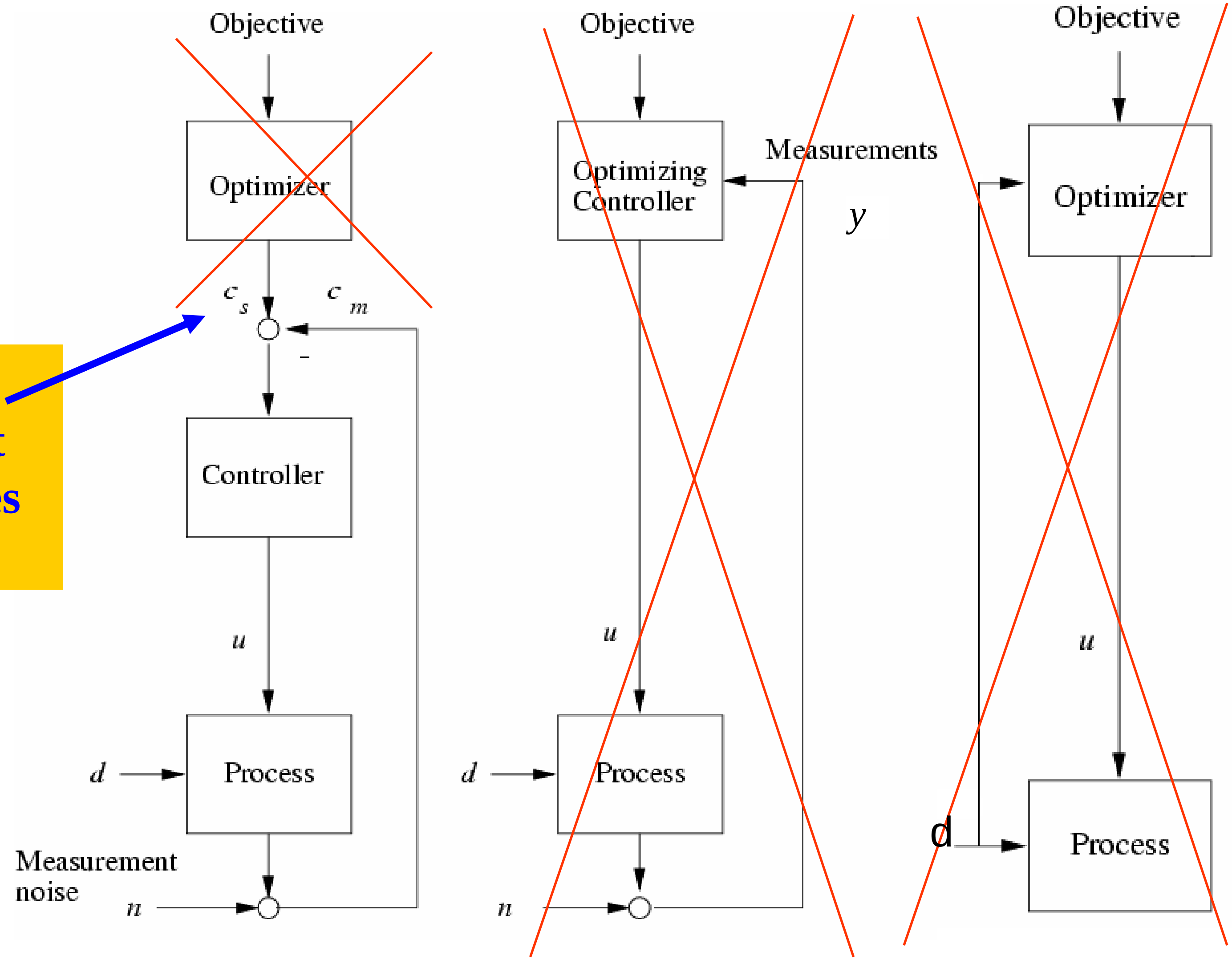
Implementation of optimal operation

- **Paradigm 1: Centralized on-line optimizing control** where measurements are used to update model and states

- **Paradigm 2:** "**Self-optimizing" control scheme** found by exploiting properties of the solution
  - Control the right variable! (CV selection)

Implementation (in practice): Local feedback control!

"Self-optimizing control:" Constant setpoints for c gives acceptable loss



Local feedback: Control c (CV)

Optimizing control

Feedforward

“self-optimizing control”

- Old idea (Morari *et al.*, 1980):

“We want to find a function c of the process variables which when held constant, leads automatically to the optimal adjustments of the manipulated variables, and with it, the optimal operating conditions.”

- **But what should we control? Any systematic procedure for finding c?**

- Remark: “Self-optimizing control” = acceptable steady-state behavior (loss) with constant CVs.

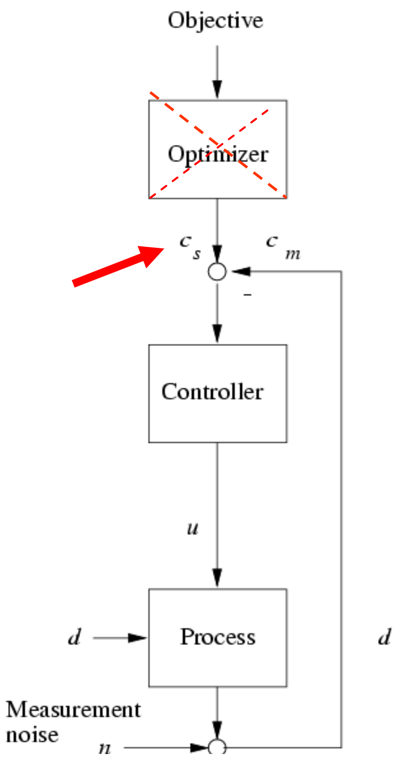
is similar to

“Self-regulation” = acceptable dynamic behavior with constant MVs.

Question: What should we control (c)?

(primary controlled variables y_1=c)

Issue:
What should we control?



- **Introductory example: Runner**

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Optimal operation of runner

– Cost to be minimized, J=T
– One degree of freedom (u=power)
– What should we control?

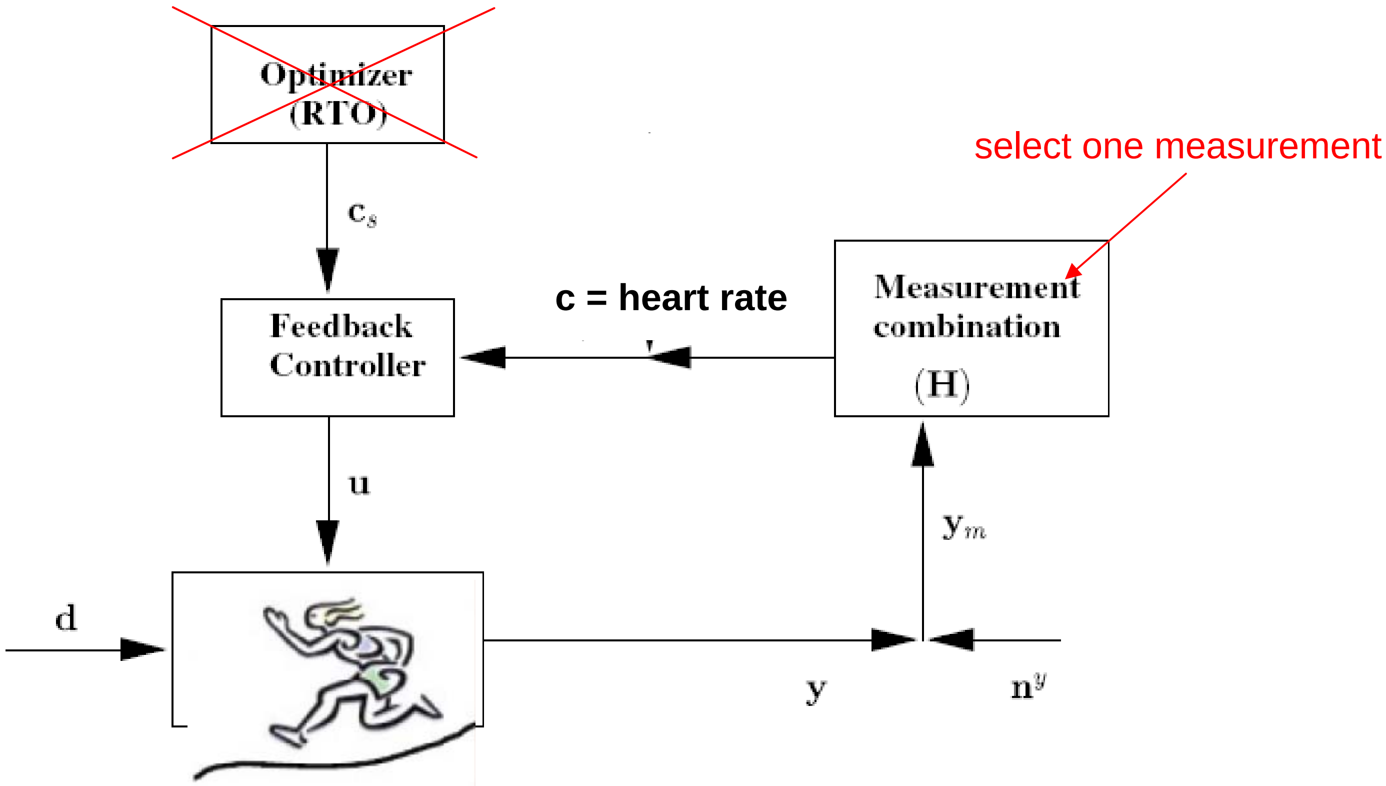
Sprinter (100m)

- 1. Optimal operation of Sprinter, J=T
  - **Active constraint control:**
    - Maximum speed (”no thinking required”)

Marathon (40 km)

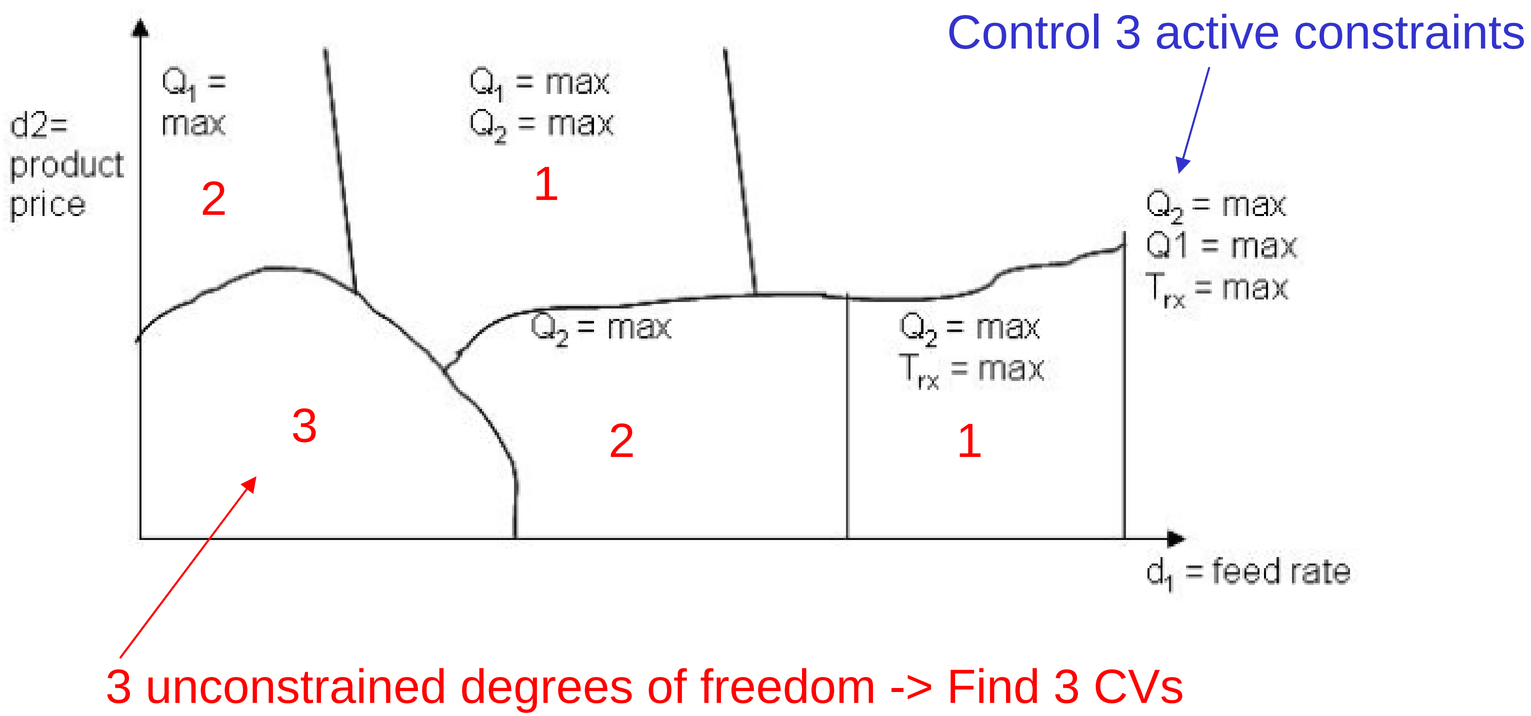
- 2. Optimal operation of Marathon runner, J=T
- Unconstrained optimum!
- Any ”self-optimizing” variable c (to control at constant setpoint)?
  - c_1 = distance to leader of race
  - c_2 = speed
  - c_3 = heart rate
  - c_4 = level of lactate in muscles

Conclusion Marathon runner



- Simple and robust implementation
- Disturbances are indirectly handled by keeping a constant heart rate
- May have infrequent adjustment of setpoint (heart rate)

Need to find new "self-optimizing" CVs (c=Hy) in each region of active constraints



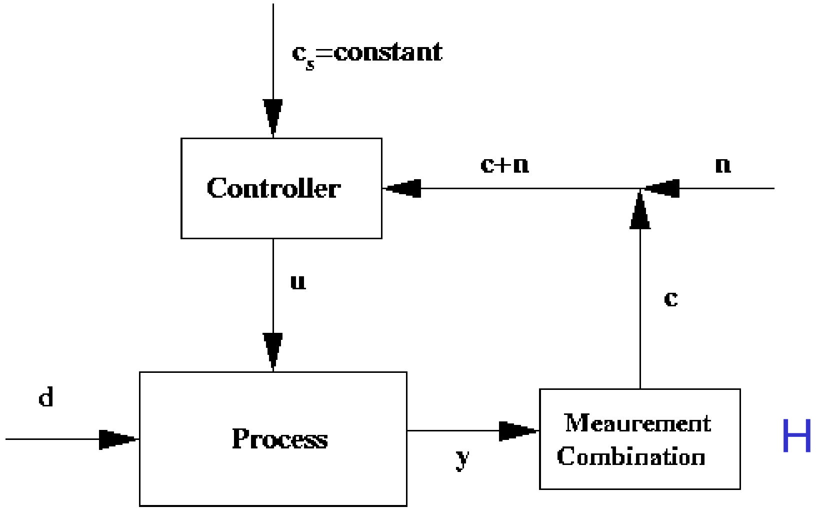
Ideal "Self-optimizing" variables

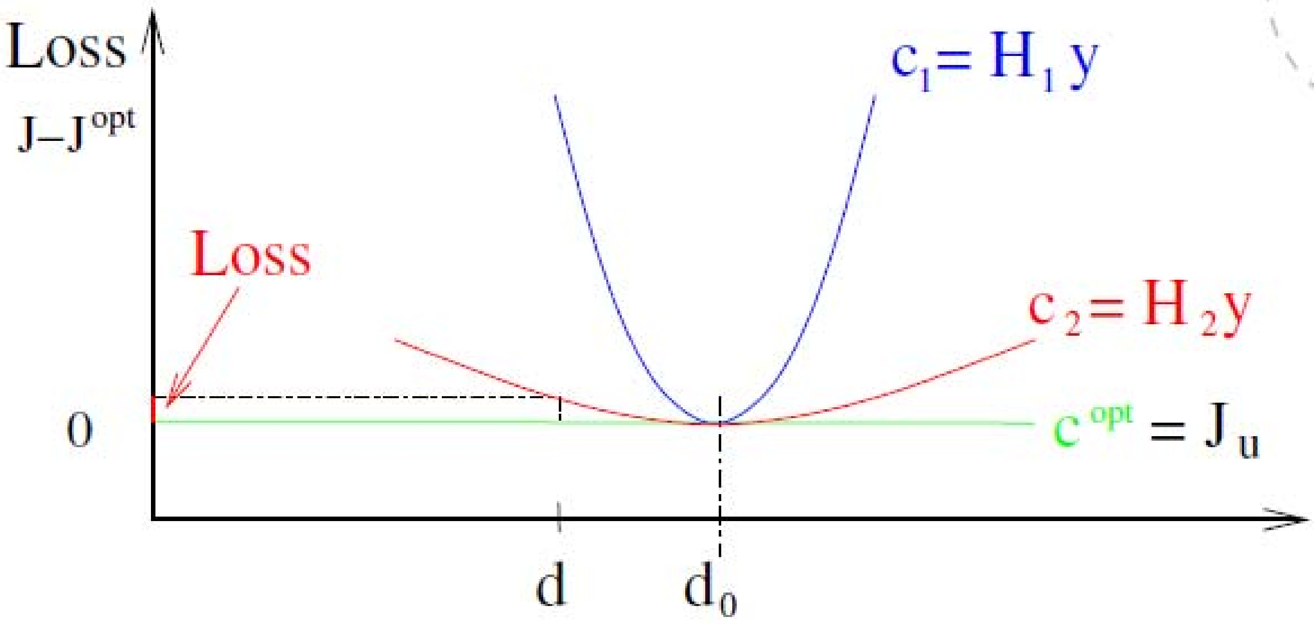
- **Operational objective: Minimize cost function J(u,d)**
- The ideal "self-optimizing" variable is the gradient (first-order optimality condition (ref: Bonvin and coworkers)):

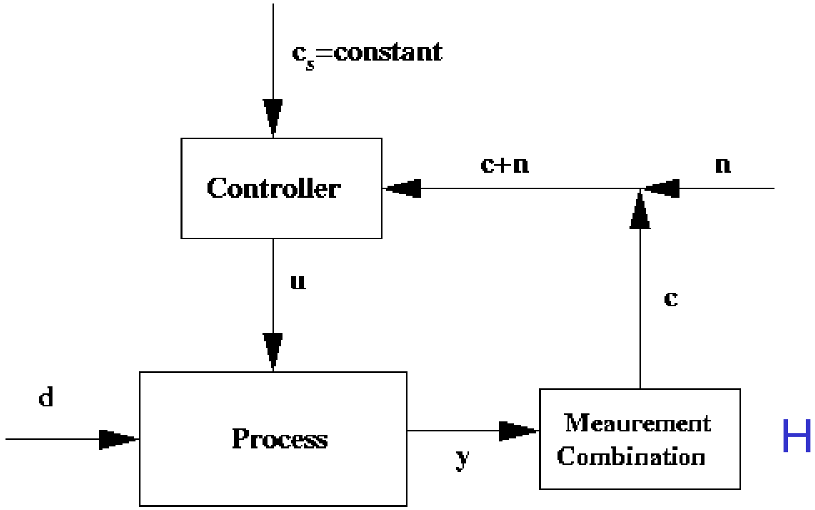
$$c = \alpha J_u; \qquad J_u = \frac{\partial J}{\partial u}$$

  - Optimal setpoint = 0
- BUT: Gradient can not be measured in practice
- Possible approach: Estimate gradient J_u based on measurements y

- **Approach here**: Look directly for c as a function of measurements y (c=Hy) without going via gradient


c_s=constant
Controller
c+n
n
u
c
d
Process
y
Meaurement Combination
H


Loss
J–J^opt
Loss
c_1= H_1 y
c_2= H_2 y
0
c^opt = J_u
d
d_0



- Single measurements:

$$\mathbf{c} = \mathbf{H}\mathbf{y} \qquad \mathbf{H} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}$$

- Combinations of measurements:

$$\mathbf{c} = \mathbf{H}\mathbf{y} \qquad \mathbf{H} = \begin{bmatrix} h_{11} & h_{12} & h_{13} & h_{14} \\ h_{21} & h_{22} & h_{23} & h_{24} \end{bmatrix}$$

Guidelines for selecting single measurements as CVs

- Rule 1: The optimal value for CV ($c=Hy$) should be insensitive to disturbances d (minimizes effect of setpoint error)
- Rule 2: c should be easy to measure and control (small implementation error n)
- Rule 3: "Maximum gain rule": c should be sensitive to changes in u (large gain $|G|$ from u to c) or equivalently the optimum J_{opt} should be flat with respect to c (minimizes effect of implementation error n)

$$\text{Maximize } \underline{\sigma}(G_s) \quad \text{where} \quad G_s = S_1 G J_{uu}^{-1/2}$$

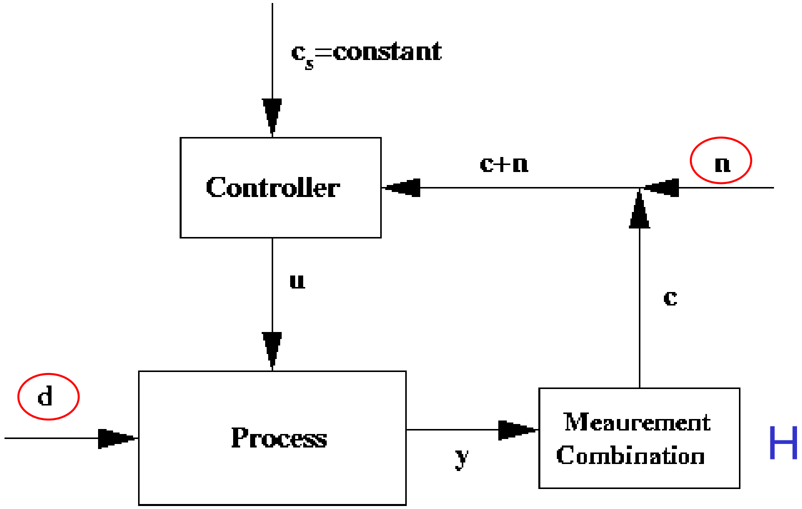
$$G = HG^y$$

Reference: S. Skogestad, "Plantwide control: The search for the self-optimizing control structure", Journal of Process Control, 10, 487-507 (2000).

Optimal measurement combination

$$\Delta c = h_1 \Delta y_1 + h_2 \Delta y_2 + \cdots = H \Delta y$$

•Candidate measurements (y): Include also inputs u



Nullspace method

Theorem

Given a sufficient number of measurements ($n_y \geq n_u + n_d$) and no measurement noise, select $\mathbf{H}$ such that

$$\mathbf{HF} = 0$$

where

$$\mathbf{F} = \frac{\partial \mathbf{y}^{opt}}{\partial \mathbf{d}}$$

Controlling $\mathbf{c} = \mathbf{Hy}$ to zero yields locally zero loss from optimal operation.

Proof nullspace method

Basis: Want optimal value of c to be independent of disturbances

$$\Rightarrow \quad \Delta c_{\text{opt}} = 0 \cdot \Delta d$$

- Find optimal solution as a function of d: $u_{opt}(d)$, $y_{opt}(d)$
- Linearize this relationship: $\Delta y_{opt} = F \, \Delta d$
- Want: $\Delta c_{opt} = H \Delta y_{opt} = HF \Delta d = 0$
- To achieve this for all values of Δ d:

$$HF = 0 \quad \Rightarrow \quad H \in \mathcal{N}(F^T)$$

- To find a F that satisfies HF=0 we must require

$$n_y \geq n_u + n_d$$

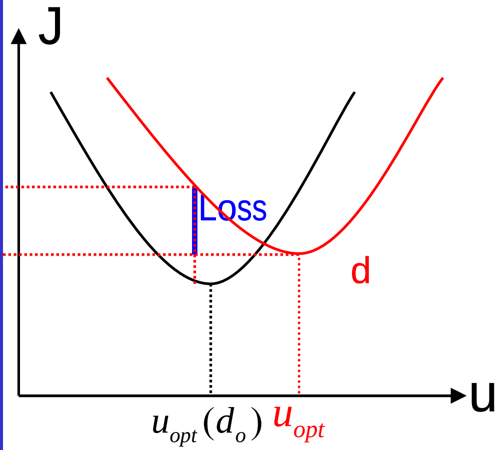
- *Optimal* when we disregard implementation error (n)

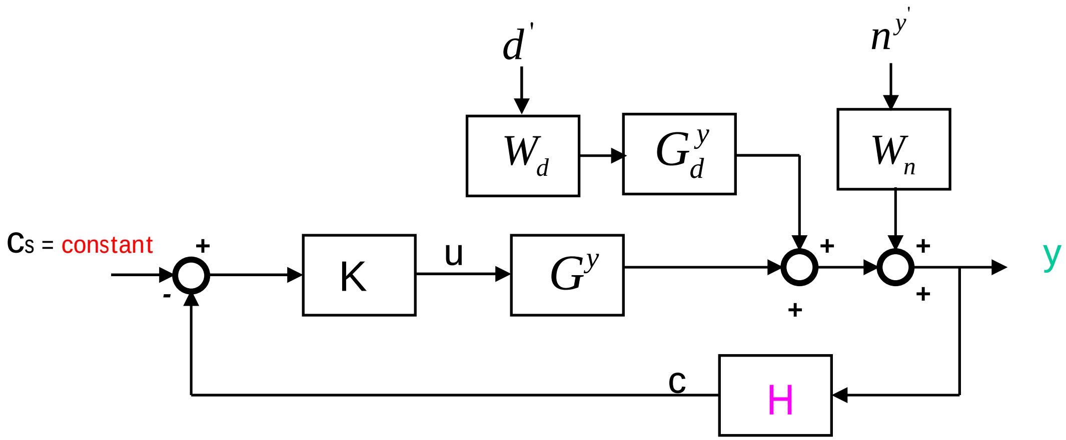
Amazingly simple!

Sigurd is told how easy it is to find H

V. Alstad and S. Skogestad, ``Null Space Method for Selecting Optimal Measurement Combinations as Controlled Variables'', Ind.Eng.Chem.Res, 46 (3), 846-853 (2007).

Generalization (with measurement noise)





Controlled variables, $c = Hy$

$$L = J(u,d) - J_{opt}(u_{opt},d)$$

$$J(u,d) = J(u_{opt},d) + J_u(u-u_{opt}) + \frac{1}{2}(u-u_{opt})^T J_{uu}(u-u_{opt}) + \zeta^3$$

$$L_{avg} = \left\| J_{uu}^{1/2}(HG^y)^{-1} HY \right\|_F^2$$

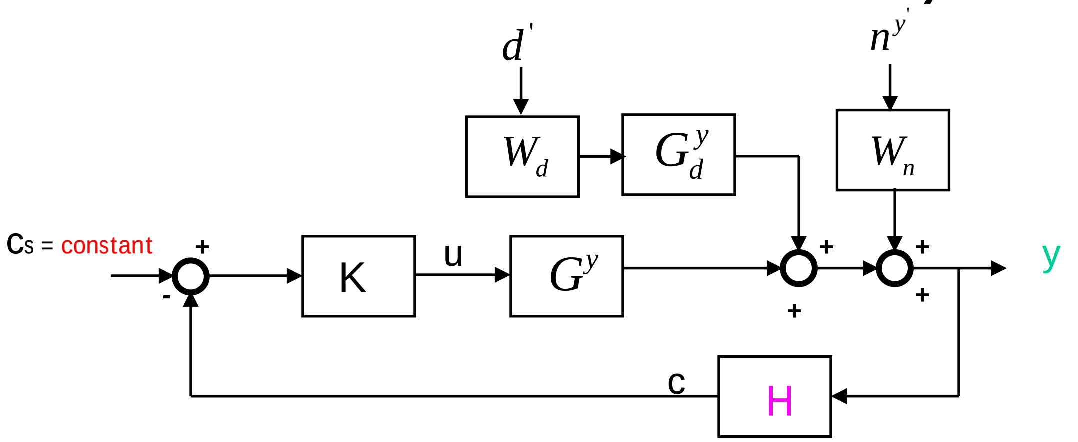
Loss with c=Hy=0 due to
(i) Disturbances d
(ii) Measurement noise n^y

$$Y = [FW_d \quad W_n],$$

$$F = G^y J_{uu}^{-1} J_{ud} - G_d^y$$

Ref: Halvorsen et al. I&ECR, 2003
Kariwala et al. I&ECR, 2008

Optimal measurement combination, c = Hy



"=0" in nullspace method (no noise)

$$\min_H \left\| J_{uu}^{1/2} (H G^y)^{-1} H [F W_d \quad W_{n^y}] \right\|_2$$

"Minimize" in Maximum gain rule
(maximize S_1 G $J_{uu}^{-1/2}$, G=HG^y)

"Scaling" S_1

$$\min_H \left\| J_{uu}^{1/2} (HG^y)^{-1} HY \right\|_F$$

$$Y = [FW_d \quad W_n]$$

Non-convex optimization problem (Halvorsen et al., 2003)

Have extra degrees of freedom

$$H_1 = DH$$ D : any non-singular matrix

$$(H_1 G_y)^{-1} H_1 = (DHG_y)^{-1} DH = (HG_y)^{-1} D^{-1} DH = (HG_y)^{-1} H$$

Improvement 1 (Alstad et al. 2009)

$$\min_H \|HY\|_F$$

$$\text{st} \quad HG^y = J_{uu}^{1/2}$$

Convex optimization problem

Global solution

Improvement 2 (Yelchuru et al., 2010)

$$\min_H \|HY\|_F$$

$$\text{st} \quad HG^y = Q$$

- Do not need J_{uu}
- Q can be used as degrees of freedom for faster solution
- Analytical solution

$$H = J_{uu}^{1/2} (G^{yT} (Y^T Y)^{-1} G^y)^{-1} G^{yT} (Y^T Y)^{-1}$$

Special case: Indirect control = Estimation using "Soft sensor"

- Indirect control: Control c= Hy such that primary output y_1 is constant
  - Optimal sensitivity $F = dy_{opt}/dd = (dy/dd)_{y1}$
  - Distillation: y=temperature measurements, y_1= product composition
- Estimation: Estimate primary variables, y_1=c=Hy
  - $y_1 = G_1 u$, $y = G^y u$
  - Same as indirect control if we use extra degrees of freedom (H_1=DH) such that $(H_1 G^y)=G_1$

$$H = J_{uu}^{1/2}((YY^T)^{-1}G^y(G^{yT}(YY^T)^{-1})^T,$$

$$Y = [FW_d \quad W_n]$$

Current research: “Loss approach” can also be used for Y = data

- More rigorous alternative to “least squares” and extensions such as PCR and PLS (Chemiometrics)

- **Why is least squares not optimal?**
  - Fits data by minimizing $||Y_1\text{-}HY||$
  - Does not consider how estimate $y_1=Hy$ is going to be used in the future.
- **Why is the loss approach better?**
  - Use the data to obtain Y, G^y and G_1 (step 1).
  - Step 2: obtain estimate $y_1=Hy$ that works best for the future expected disturbances and measurement noise (as indirectly given by the data in Y)

$$H = J_{uu}^{1/2}((YY^T)^{-1}G^y(G^{yT}(YY^T)^{-1})^T,$$

$$Y = [FW_d \quad W_n]$$

Toy Example

$J = (u - d)^2$

$n_u = 1$ unconstrained degrees of freedom

$u_{\text{opt}} = d$

Alternative measurements:

$y_1 = 0.1(u - d)$

$y_2 = 20u$

$y_3 = 10u - 5d$

$y_4 = u$

Scaled such that:

$|d| \leq 1$, $|n_i| \leq 1|$, i.e. all y_i's are ± 1

Nominal operating point:

$d = 0 \Rightarrow u_{\text{opt}} = 0, y_{\text{opt}} = 0$

What variable c should we control?

Reference: I. J. Halvorsen, S. Skogestad, J. Morud and V. Alstad, "Optimal selection of controlled variables", Industrial & Engineering Chemistry Research, 42 (14), 3273-3284 (2003).

Toy Example: Single measurements

A. Maximize minimum singular value, $|G_s|$

c	G	Expected variation in y $y_{\text{span}} = \lvert y_{\text{opt}}\rvert + \lvert n \rvert$	$\lvert G_s \rvert = \lvert G \rvert / y_{\text{span}}$	Rank
y_1	0.1	$0 + 1 = 1$	$0.1/1 = 0.1$	4
y_2	20	$20 + 1 = 21$	$20/21 = 0.95$	2
y_3	10	$5 + 1 = 6$	$10/6 = 1.67$	1
y_4	1	$1 + 1 = 2$	$1/2 = 0.5$	3

Loss = constant / $|G_s^2|$

C. Exact evaluation of loss: Same order

$$L_{wc,1} = 100$$
$$L_{wc,2} = 1.0025$$
$$L_{wc,3} = 0.26$$
$$L_{wc,4} = 2$$

Constant input, c = y_4 = u

Want loss < 0.1: Consider variable combinations

Toy Example

C. Optimal combination

Need two measurements. Best combination is y_2 and y_3:

$$\begin{pmatrix} y_2 \\ y3 \end{pmatrix} = \begin{pmatrix} 20 & 0 \\ 10 & -5 \end{pmatrix} \begin{pmatrix} u \\ d \end{pmatrix}; \ \underline{\sigma} = 4.45$$

Optimal sensitivity:

$$y_{\text{opt}} = Fd; F = \begin{pmatrix} 20 \\ 5 \end{pmatrix}$$

Optimal combination:

$$HF = 0 \Rightarrow \begin{pmatrix} h_1 & h_2 \end{pmatrix} \begin{pmatrix} 20 \\ 5 \end{pmatrix} = 0 \Rightarrow 20h_1 + 5h_2 = 0$$

Select $h_1 = 1$. Get $h_2 = -20h_1/5 = -4$, so

$$c_{\text{opt}} = y_2 - 4y_3$$

Check: $c = y_2 - 4y_3 = 20u - 40u + 20d = -20(u-d)$ *(OK!)*

Note: The scaled gain for $c = y_2 - 4y_3$ is $|-20|/(0+5) \cdot \sqrt{2} = 2.83$. Best so far!

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Conclusion

- Systematic approach for finding CVs, c=Hy
- Can also be used for estimation

$$\min_H \| J_{uu}^{1/2} (HG^y)^{-1} H \underbrace{[F W_d \quad W_{n^y}]}_{Y} \|_2$$

S. Skogestad, ``**Control structure design for complete chemical plants'',** *Computers and Chemical Engineering*, **28** (1-2), 219-234 (2004).

V. Alstad and S. Skogestad, ``Null Space Method for Selecting Optimal Measurement Combinations as Controlled Variables", *Ind.Eng.Chem.Res*, **46** (3), 846-853 (2007).

V. Alstad, S. Skogestad and E.S. Hori, ``Optimal measurement combinations as controlled variables", *Journal of Process Control*, **19**, 138-148 (2009)

Ramprasad Yelchuru, Sigurd Skogestad, Henrik Manum , MIQP formulation for Controlled Variable Selection in Self Optimizing Control *IFAC symposium DYCOPS-9*, Leuven, Belgium, 5-7 July 2010

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